Asynchronous Datapath Synthesis with Statistical Schedule Length Analysis

Koji OHASHI
School of Information Science,
Japan Advanced Institute of Science and Technology

As a VLSI system becomes larger and the clock period becomes shorter, it becomes difficult to control a digital circuit by a global clock under the fluctuation of datapath delay and clock skew. Asynchronous design is considered as a promising alternative, since it is free from such a global clock. Also it has the potential to achieve low power consumption, higher average-case performance, and higher reliability [1].

To design a cost effective high performance asynchronous system for a specified application, optimization of datapath in register transfer level is an important design step. Scheduling and resource binding are major subtasks in datapath synthesis not only for synchronous systems but also for asynchronous systems. Several synthesis systems for asynchronous systems have been proposed [3, 4]. In these systems, the execution time of each operation is treated as variable, and "resource edges" [3] or "disjunctive arcs" [4] are added to an input data flow graph in scheduling phase. In evaluation phase (of a schedule and a datapath), they introduce three different constant delays for the execution time of each operation: minimum (or best-case) delay, typical delay, and maximum (or worst-case) delay. Then, under the assumption that the execution time of all operations are typical delays (minimum delays, maximum delays), the longest path length of that graph is calculated, and we obtain typical (minimum, maximum) total computation time of an application for the schedule and the datapath.

When delays of all component vary uniformly, the above constant delay model seems to be an acceptable way to handle delay variations. However, for random delay variations due to local supply noise, local variations of temperature, cross talk between wires, local manufacturing imperfections, etc., the calculation of minimum, typical, and maximum total computation time based on the constant delay model is unacceptable.

In order to handle these random delay variations, in this research, we propose statistical schedule length analysis method for evaluating schedule and datapath during asynchronous datapath synthesis. The execution time of each operation is modeled by a stochastic variable having normal distribution, as usually assumed in a statistical analysis of a combinatorial circuit. An algorithm to calculate the distribution of total computation time of an application algorithm under given schedule and resource assignment is presented. The proposed statistical analysis handles three correlations; (1) correlation between delays on different modules and nets, (2) structural correlation (re-convergent paths in a scheduling graph), and (3) correlation induced by resource sharing (depends on resource binding).

We consider the problem to find mean $E[l(o_{quit})]$ and variance $V[l(o_{quit})]$ of $l(o_{quit})$ of the sink node o_{quit} in a scheduling graph G_S, where $l(v)$ be the longest path length from the source node o_{init} to a node v in G_S. The proposed algorithm to compute the mean $E[l(v)]$ and the variance $V[l(v)]$ of $l(v)$ is summarized as follows.

1. All nodes in the scheduling graph G_S are sorted in topological order.
2. If $l(o_{quit})$ is computed, then $E\left[l(o_{quit})\right]$ and $V\left[l(o_{quit})\right]$ are outputted. Otherwise, a node v is selected from G_S in topological order.
3. If $v = o_{init}$, we set $E[l(v)] = V[l(v)] = 0$ and $R[l(v), l(v)] = 1$, and go to 2. Otherwise, for each arc $e_i = (u_i, v)$ $(i = 1, 2, \cdots, f(v))$, where $f(v)$ is the number of incoming arcs of v, we consider the longest path length from o_{init} to v passing through e_i, which is denoted by $l_i^t(v)$. We calculate the mean $E[l_i^t(v)]$ and the variance $V[l_i^t(v)]$ of $l_i^t(v)$.
   Next, for a node $x \in V_S$ whose $l(x)$ has been already computed, we calculate correlation coefficient $R\left[l_i^t(v), l(x)\right]$ between $l_i^t(v)$ and $l(x)$. Note that we set $R[l(x), l(u_i)] = 1$, if $x = u_i$.
   Similarly, for each arc $y \in A_S$ we calculate correlation coefficient $R\left[l_i^t(v), w(y)\right]$ between $l_i^t(v)$ and $w(y)$.
4. Let $l_i(v)$ be the longest path length from o_{init} to v passing through any arc $e_j = (u_j, v)$ $(j = 1, 2, \cdots, i(\leq f(v)))$. Instead of $l_i(v) = \max[l_1^t(v), l_2^t(v), \cdots, l_i^t(v)]$, we recursively calculate $l_i(v) = \max\left[l_{i-1}(v), l_i^t(v)\right]$ if $i \geq 2$, otherwise $l_i(v) = l_i^t(v)$.
   The mean $E[l_i(v)]$ and the variance $V[l_i(v)]$ of $l_i(v) = \max\left[l_{i-1}(v), l_i^t(v)\right]$ are calculated. Also the correlation coefficients $R[l_i(v), l(x)]$ and $R[l_i(v), w(y)]$ between $l_i(v) = \max\left[l_{i-1}(v), l_i^t(v)\right]$ and $l(x)$ of each node x and $w(y)$ of each arc y, respectively, are calculated.
5. $l(v) = l_{f(v)}(v)$ is computed as a stochastic variable with a normal distribution with the mean $E[l(v)] = E[l_{f(v)}(v)]$ and the variance $V[l(v)] = V[l_{f(v)}(v)]$. The correlation coefficients $R[l(v), l(x)]$ and $R[l(v), w(y)]$ between $l(v)$ and $l(x)$ of each node x and $w(y)$ of each arc y, respectively, are also computed, and go to 2.

The proposed statistical analysis algorithm is implemented using C program language on a 1GHz Pentium III personal computer, and is applied to three datapaths and schedules (*Datapath A, Datapath B,* and *Datapath C*), which are synthesized in [4]. The delays of adders and multipliers are modeled by normal distributions with $N(9, 0.11)$ and $N(17, 1.00)$, respectively. If two operations o_i and o_j are assigned to the same functional unit, we set the correlation coefficient $\rho(e_i, e_j) = 1$ between $w(e_i)$ and $w(e_j)$ for two arcs $e_i = (o_i^s, o_i^e)$ and $e_j = (o_j^s, o_j^e)$ in a scheduling graph G_S. It is assumed that the other weights of arcs are independent.

Table 1 shows the results of our statistical analysis and Monte Carlo simulation (100000 iterations). In the columns indicated by E and $\sqrt{V}$, the means and the standard deviations of the distributions of total computation time are shown, respectively. The relative errors to the results of Monte Carlo simulation are also shown in the same table. As we can see from this table, our method provides the mean and the standard deviation with the relative errors 0.16 % and -3.87 %, respectively, on average.

Table 1: Total computation time for datapaths and schedules in [4].

	Ours			Monte Carlo			Error [%]	
	E	$\sqrt{V}$	time [s]	E	$\sqrt{V}$	time [s]	E	$\sqrt{V}$
Datapath A	153.23	3.02	0.11	153.00	3.16	2.75	+0.15	-4.43
Datapath B	154.10	3.44	0.15	153.77	3.67	2.80	+0.21	-6.27
Datapath C	184.46	6.57	0.15	184.26	6.63	2.79	+0.11	-0.90
average							+0.16	-3.87

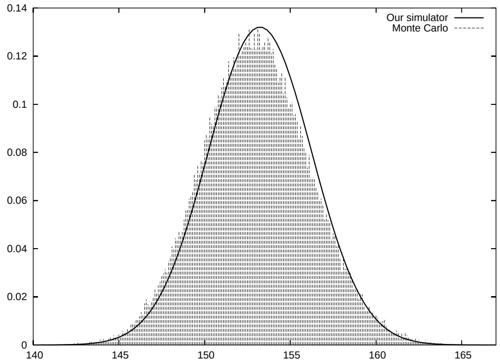

Figure 1: The distribution of total computation time for *Datapath A*.

To compare the shape of probability density function, we compute a large number of samples of total computation time for *Datapath A* using Monte Carlo simulation, and the results are shown in Fig. 1 as a histogram of total computation time. The solid curve in the same figure represents a perfect normal distribution with the mean and standard deviation obtained from our statistical analysis. From the figure, we can see that our result is very close to the distribution obtained by Monte Carlo simulation.

Next, we consider the problem to find a datapath and a schedule with minimum mean total computation time (i.e., minimum $E[l(o_{quit})]$) with maximum total computation time $T_{max} \leq T_M$ under given set of available modules and a constant T_M (the upper bound of maximum total computation time T_{max}). Note that T_{max} is computed using maximum execution time of each operation. To find such datapath and schedule, we incorporate the proposed statistical schedule length analysis into binding exploration based synthesis system [4], and set $E[l(o_{quit})]$ for the objective function to be minimized. Three datapath synthesis benchmarks; differential equation solver, four-order Jaumann wave digital filter, and fifth-order elliptic wave filter are used as target algorithms. For comparison purpose, datapaths and schedules are also synthesized by using the conventional objective function (minimize typical total computation time $T_{typical}$) used in [3, 4]. Synthesis results are evaluated by Monte Carlo simulation, and means and standard deviations of the total computation time are obtained.

In these experimental results, our system using the proposed statisitcal schedule length analysis always provides better solutions. For the case of small target algorithms or small number of functional units, the performance difference between datapaths designed by our system and conventional one seems not so large. However, our system tends to generate better solutions than conventional one in the mean total computation time, when the size of a target algorithm becomes larger, the number of functional units becomes larger, and the variance of execution delay of each module becomes larger.

The normal distribution is not always adequate for the delay analysis of asynchronous datapaths. Development of a proper model and algorithms to compute statistical parameters, which reflect practical random delay variations, are left for future work.

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