Efficient extraction of minimal inconsisntent sets from a legal knowledge(The report of GRP)

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The Aim of the Research Judicial knowledge is a conglomeration of laws, facts, and precedent cases, and can be modeled by a large set of logical propositions. However, arbitrary expansion of knowledge, such as legislation of new laws or addition of cases, may cause inconsistency from the inside. In addition, when natural language expression for the legal knowledge is translated into logical terms, it may include subjective interpretation that can be a barrier for sound inference. It is ideal if we could manage to keep consistency of the knowledge base for any revision, expansion, and subtraction of a piece of knowledge; however, from a practical point of view, the maintenance seems too difficult. It may be better for us to admit that the knowledge may include some kind of discrepancy, though we can construct a consistent argument as its subset. Therefore, we construct an algorithm which extracts minimal consistent sets from a knowledgebase.

An Approach and an Idea In logical representation of legal knowledge, which consists of inference rules, facts, and arbitrary interpretations may include inconsistent propositions. In this study, we propose an efficient algorithm to extract minimal inconsistent sets out of a legal knowledge, that could help reasoning of a legal agent. In order to reduce the computational complexity, we divide the algorithm into the following two steps. Firstly, we divide the knowledge into multiple cliques, each of which consists of subformulae of a certain formula, to decrease the size. Secondly, we extract arguments, that is a chain of inferences, from a clique. Then, we merge several arguments in conflict with each other. Hence, we acquire the minimal inconsistent sets.

As has often been discussed in defeasible reasoning, the longer the chain of inferences is, the stronger the argument can be regarded[1]. Thus, extracting a larger consistent set out of a knowledge is an important goal for legal reasoning. In this study, we present our effort in this field, regarding the judicial knowledge as a set of logical propositions. However, generally speaking, deciding a maximal consistent set is reduced to a notorious NP-complete problem MAX-k-SAT[2], though we can only extract an approximation of the consistent set. On the contrary, we can expect it is rather computationally less complex to find an inconsistent set. Because judicial arguments involve acts of finding inconsistency in the reasoning of opponent, the *minimal inconsistent set* would be worth considering. We will show an algorithm that extracts a minimal inconsistent set in moderately sizable complexity. The existence of inconsistency in a knowledge set implies that there can be two opposing arguments, each of which is a chain of implications. We reform legal knowledge in rules and facts, and regard an argument as a pair of a chain of implication and a consequence, and then search for an inconsistency.

In this study, we restrict the form of propositions and will find a clique that consists of a certain group of proposition variables. In which, we search for

[1] John L. Pollock: How to reason defeasibly, In *Artificial Intelligence*, Vol.57, pp. 1–42, 1992

[2] Michael R. Garey and David S. Johnson: Computers and Intractability, In *A Guide to the Theory of NP-Completeness*, W. H. Freeman & Co,1979

minimal inconsistent sets, instead of maximal consistent sets; so that the reasoner of legal arguments would construct longer chains to support some consequence, taking care not to include those minimal inconsistency sets.

Here, we give a definition of Minimal inconsistency.

Theorem 1 *A Minimal Inconsistent SetLet Σ be sets of formulae. Γ is able to denote a minimal inconsistent set in Σ iff $\Gamma \vdash \bot$, and $\forall\phi[\phi \in \Gamma, \Gamma - \{\phi\} \nvdash \bot]$.*

The Progress of the Year Our progress is that we extracted minimal inconsistent sets from a knowledge which is expressed 2CNF[3] on the graph theory without provers. We defined a normal form for above as following:

Theorem 2 *Normal Form of Implication: Let Σ be knowledge, ϕ be element in Σ, C be clause, R be rule or fact, and p be literal. The normalized knowledge base is: $\Sigma = \bigcup_i \phi_i$, $\phi_i = \bigwedge_j R_j^{\phi_i}$, $R_j^{\phi_i} = p_k \to p_l$. where i, j, k, l are arbitrary natural numbers. Especially, $\top \to p_k$ denotes a fact, where $\top$ is a constant proposition that is always evaluated as true. The form is equivalent to 2CNF.*

We can consider the knowledge bese as a graph. Thus, using our algorithm, we can find minimal inconsistent sets without a prover. The algorithm that extracts a set of minimal inconsistent sets consists of three parts. The first part is to extract all the *upper* and *lower* paths of a node in a graph where:

Theorem 3 *Upper/lower path An upper path of v is a path that starts from v, to the furthest accessible node, with regard to the direction of edges. A lower path of v is a path that ends up with v.*

A literal that appears in a path is implied by another literal in the lower part of the path. These paths are used to classify the three kinds of inconsistency(Conclusion, Premise, and Argument inconsistency), in the second part to find proved literals. The third part is to combine two conflictive arguments. In the above, we got some paths of proof for the literals. Those paths are arguments. Then, the program exhaustively combines the arguments that concludes v with those concludes $\neg v$, i.e., $\{Arg_{i1}, Arg_{i2}, Arg_{i3}, \cdots\} \vdash v$ is combined with $\{Arg_{j1}, Arg_{j2}, Arg_{j3}, \cdots\} \vdash \neg v$, and produces: $\{\{Arg_{i1}, Arg_{j1}\}, \{Arg_{i2}, Arg_{j2}\}, \{Arg_{i3}, Arg_{j3}\}, \cdots\}$.

[3] Two Conjactive Normal Form

The Future Direction In this research, we constructed an algorithm which extracts minimal inconsistent sets from a legal knowledge-base, which may include inconsistency in general. The sets can contribute to legal reasoning, such as opposing arguments and concequences. Currently, our algorithm can only process 2CNF, that is effective in finding opposing arguments, though legal knowledge would not be expressed by 2CNF completely. Thus, we need a new algorithm which can extract minimal inconsistent sets from a knowledge-base which contains arbitrary formulae, provided that elements of the knowledge-base is well-formed.

When a legal knowledge-base is inspected for inconsistency, a computer system cannot necessarily discover it. For example, a knowledge-base has a rule of “Vehicles are not permitted to enter the park” and a fact of “A car entered the park.” We can consider that the case is illegal. However, a computer system cannot recognize it, since it cannot unify ‘vehicle’ and ‘car’. Thus, we need a logical framework which can find that ‘vehicle’ is a superordinate concept of ‘car’. Because we can expect that the *order-sorted* logic could solve the problem, we should consider adopting the logic into our algorithm.

The minimum inconsistency set in this research has the concept of graded negation[4] as one of the logical frameworks for solving inconsistency. Therefore, we also consider adopting adopt it and can present *paraconsistent* logic of legal reasoning.

The List of the Publication, and Systems

- Efficient extraction of minimal inconsisntent sets from a legal knowledge, ICAIL05 (under submission)
- Extraction of minimal inconsisntent sets from a legal knowledge, JSAI2005(under submission)
- Extraction of minimal inconsistent set on the graph theory, COMPUTATIONAL INTELLIGENCE CI 2005(in preparation)

[4] D.Gabbay and Hunter: Negation and Contradiction, in What is negation?, Kluwer Publishers, pp. 89–100, 1999