Introduction to Algorithms and Data Structures

Searching (1): Sequential search and its analysis

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Main topic:

SEARCH PROBLEM

Search Problem

- Problem: *S* is a given set of data. For any given data *x*, determine efficiently if *S* contains *x* or not.

- Efficiency: Estimate the time complexity by $n = |S|$, the size of the set *S*
  - In this problem, "checking every data in *S*" is enough, and this gives us an upper bound O(n) in the worst case.
  - Can we do better?
  - How about *dictionary*?

How to tackle the problem

- Consider data structure and how to store data
  - Data are in an array in any ordering
  - Data are in an array in increasing order
- Search algorithm: The way of searching
  - Sequential search
  - m-block method
  - Double m-block method
  - Binary search
- Analysis of efficiency

We introduce these methods to explain our naïve idea.

Data structure 1
Data are stored in arbitrary ordering

- Each element in the set *S* is stored in an array s from s[0] to s[*n*-1] in any arbitrary ordering.

s[ ]=									
	37	12	25	9	87	33	65	3	29

Sequential search

- Input: any natural number *x*
- Output:
  - If there is i such that s[i] == *x*, output i
  - Otherwise, output -1 (for simplicity)

```
for (i=0; i<n; ++i)
   if(x==s[i]) return i;
return -1;
```

In the worst case, we need *n* comparisons.
Thus, the running time is proportional to *n*.
→ O(*n*) time algorithm

Example: Real code of seq. search

```
public class i111_03_p7{
    public static void Main(){
        int[] data = new int[]{37,12,25,9,87,33,65,3,29};
        int len = data.Length;

        int target = 87;
        int result = find(target,len,data);
        if (result == -1) {
            System.Console.WriteLine(target+" not found");
        } else {
            System.Console.WriteLine(target+" is at index "+result);
        }
    }

    static int find(int x, int n, int[] s) {
        for (int i=0; i<n; i++) {
            System.Console.Write(i+" ");
            if (x==s[i]) return i;
        }
        return -1;
    }
}
```

Precise time complexity of sequential search

- At most 3n + 2 steps

Initialization of i takes 1 operation

```
for (i=0; i<n; ++i)
    if(x==s[i]) return i;
return -1;
```

For the number of loops ≦ n,

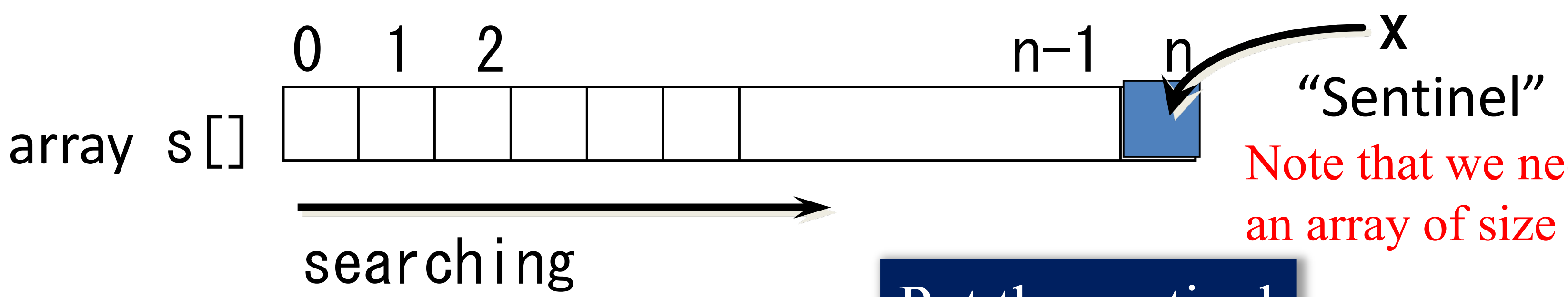
comparison × 2 (==, <)

increment × 1 (++)

Return takes 1 operation

Programming tips 1: simplify by using “sentinel”

Before searching, push *x* itself at the end of the array;
Then you definitely have x==s[i] for some 0<=i<=n
So you do not need the check i<n any more.



Note that we need an array of size n+1

```
s[n] = x;
i = 0;
while(x != s[i])
 i = i+1;
if(i < n) return i;
else      return -1;
```

Put the sentinel

Simple loop!
→ 2 operations

At most 2n+4 (<3n+2) operations $=O(n)$

Analysis of the number of comparisons

```
s[n] = x;
i = 0;
while(x!=s[i])
 i = i+1;
if(i < n)
  return i;
else
  return -1;
```

Consider best/worst/average cases

- The best case: 1
  - when s[0] == x
- The worst case: n
  - when x is not in s[0]…s[n-1]
- The average case : $\sum_{i=1}^{n+1} \frac{i}{n} = \frac{n+2}{2}$
  - The expected value of # of comparisons
  - The i-th element is compared with probability 1/n
  - The number of comparisons when x is equal to the i-th element is i.

※average is close to *n* when we often have the case that *x* is not in data
※It depends on the situation that which case is important

What happens if we use “nice” data structure?

Data structure 2

Data in the array in increasing order

We don't consider how can we do now

- s[]=

3	9	12	25	29	33	37	65	87	*x*

- Q: Any improvement in sequential algorithm?

```
s[n]=x;
i = 0;
while(x!=s[i])
 i = i+1;
if(i < n) return i;
else      return -1;
```

Idea

We can stop when s[i] is greater than x

x!=s[i] → x>s[i]

Data structure 2

Data in the array in increasing order

We don't consider how can we do now

- s[]=

3	9	12	25	29	33	37	65	87	*x*

- Q: Any improvement in sequential algorithm?

```
s[n]=x;
i = 0;        It does not happen over x!
while(s[i]<x)
 i = i+1;
if(i < n) return i;
else      return -1;
```

Idea

We can stop when s[i] is greater than x

x!=s[i] ➔ x>s[i]

Data structure 2

Data in the array in increasing order

- s[]=

3	9	12	25	29	33	37	65	87	*x*

- Q: Any improvement in sequential algorithm?

```
s[n]=x;
i = 0;
while(s[i]<x)
 i = i+1;
if(i < n) return i;
else      return -1;
```

We can stop when s[i] is greater than x
x!=s[i] → x>s[i]

It may stop even if i<n
i<n → s[i]==x
E.g, if x=30, we have i<n (5<9) but it should return (−1)

Look!

Data structure 2

Data in the array in increasing order

- s[]=

3	9	12	25	29	33	37	65	87	*x*

- Q: Any improvement in sequential algorithm?

```
s[n]=x;
i = 0;
while(s[i]<x)
 i = i+1;
if(s[i]==x) return i;
else        return -1;
```

We can stop when s[i] is greater than x

`x!=s[i]` → `x>s[i]`

It may stop even if `i<n`

`i<n` → `s[i]==x`

Much intuitive condition!

Data structure 2

Data in the array in increasing order

- s[]=

3	9	12	25	29	33	37	65	87

x

Look!

- Q: A ... ential algorithm?

When x is not in s[], it returns n
s[n]=x ➔ s[n]=x+1

```
s[n]=x;
i = 0;
while(s[i]<x)
 i = i+1;
if(s[i]==x) return i;
else        return -1;
```

We can stop when s[i] is greater than x
x!=s[i] ➔ x>s[i]

It may stop even if i<n
i<n ➔ s[i]==x

Data structure 2
Data in the array in increasing order

- s[]=

3	9	12	25	29	33	37	65	87	*x*+1

- Q: A … equential algorithm?

When x is not in s[], it returns n
s[n]=x ➔ s[n]=x+1

```
s[n]=x+1;
i = 0;
while(s[i]<x)
 i = i+1;
if(s[i]==x) return i;
else        return -1;
```

We can stop when s[i] is greater than x
x!=s[i] ➔ x>s[i]

It may stop even if i<n
i<n ➔ s[i]==x

Data structure 2
Data in the array in increasing order

- s[]=

3	9	12	25	29	33	37	65	87	*x*+1

  - Exit from loop when: s[i]≧x
  - Check after loop: s[i]==x
  - Sentinel: greater than x, e.g., x+1

```
s[n]=x+1;
i = 0;
while(s[i]<x)
 i = i+1;
if(s[i]==x) return i;
else        return -1;
```

Q. Improve of comparison?

A. Average is better.
But the same in
the worst case

Q : When the average is better?

Example: Real code of seq. search in increasing order

```
public class i111_03_p18{
    public static void Main(){
        int[] data = new int[]{3,9,12,25,29,33,37,65,87,-1};
        int len = data.Length-1;

        int target = 17;
        int result = find(target,len,data);
        if (result == -1) {
            System.Console.WriteLine(target+" not found");
        } else {
            System.Console.WriteLine(target+" is at index "+result);
        }
    }

    static int find(int x, int n, int[] s) {
        s[n] = x+1;
        int i=0;
        while (s[i]<x) {
            System.Console.Write(i+" ");
            i++;
        }
        if (x==s[i]) return i;
        return -1;
    }
}
```

Minor improvements of number of comparisons in sequential search

(Tips 1)

In the array, the minimum data is the first, and the maximum data is the last. Thus, depending on x and them,
we can change the direction of search.

➔ We still need n-1 comparisons in the worst case

(Tips 2)

First, compare x with the medium data s[n/2]. If x is larger, search the right half, and search the left half otherwise.

➔At most n/2 comparisons. Much smaller.

➔It is still $O(n)$, but,,,

Drastic improvement from $O(n)$!!