Introduction to Algorithms and Data Structures

0. Introduction to Introduction to Algorithms and Data Structures

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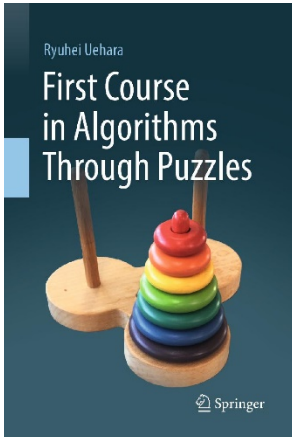
Summary

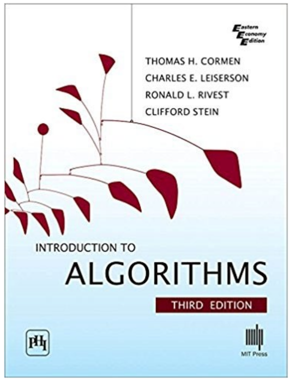
Goal: To understand the meaning and importance of algorithms.

A formal procedure for solving a problem is called an algorithm and a way of storing data in a computer is called a data structure. There may be a number of combinations of algorithms and data structures for a problem, in general. It is important to evaluate them by computation time and space requirement to choose the best combination. In this lecture, a general but basic scheme for algorithm design through validation of the correctness of algorithms and investigation of improvement of algorithm efficiency is explained.

References

- Textbooks
  - "First Course in Algorithms through Puzzles," Ryuhei Uehara, 2019, Springer.
  - "Introduction to Algorithms, 3rd ed." Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest, Clifford Stein, 2010, MIT Press.





We do not necessarily follow the textbooks,,,

Evaluations

- Viewpoint of evaluation :
  - Comprehension of theory and implementation of algorithms and data structures.
- Evaluation method :
  - Reports
    - I will ask small reports each day.
    - I will prepare big report problems, which will be distributed on January 9.
    - Summary of a lecture on January 10.
    - Submit your report to Prof. Wint Thida Zaw (wintthidazaw@uit.edu.mm)

Schedule of Lectures (1)

January 7: 10:00-12:00 and 13:00-15:00

0. Intruduction to Introduction to Algorithms
1. Foundation of Algorithms (1): Basic models
2. Foundation of Algorithms (2): Simple Basic Algorithms
3. Searching (1): Sequential Search and its analysis
4. Searching (2) Block Search

January 8: 10:00-12:00 and 13:00-15:00

5. Searching (3) : Binary Search and Hash method
6. Data Structure (1): Stack, Queue, and Heap
7. Data Structure (2): Binary Search Tree and (its balancing)
8. Sorting (1): Bubble sort, Insertion sort, and Heap sort

January 9: 10:00-12:00 and 13:00-15:00

9. Sorting (2): Merge Sort, Quick sort, complexity of sort algorithms, and counting sort
10. Data Structure (4): Data structures for graphs
11. Graph Algorithms: Breadth-first search and depth-first search
12. Advanced Algorithm: Dynamic Programming

January 10: Special lectures on recent algorithms by the following professors

Schedule of Lectures (2)

January 10: Special lectures on recent algorithms by the following professors

- Spanning trees and Cotrees in Digraphs
  - Prof. Muhammad Kaykobad, Bangladesh University of Engineering and Technology
- Graph Drawing
  - Prof. Md. Saidur Rahman, Bangladesh University of Engineering and Technology
- Approximation Algorithms using ILP
  - Prof. Subhas Nandy, Indian Statistical Institute
- Dispersion Problems
  - Prof. Shin-ichi Nakano, Gunma University
- Computational Origami
  - Prof. Ryuhei Uehara, Japan Advanced Institute of Science and Technology

A survey of some talk(s) you prefer will be a part of the report.

Requirements

- Lectures are given in English
- You can ask/answer in English (or Japanese :-)

- Note that “algorithm” and “programming” are different. “programming” is implementation of algorithm.

- We *do not* assume any specific language, but we use C as an example.

- You can use any programming language such as c, C++, Java, Delphi,,,, perl, ruby, python, basic… in your reports. (You can also give in pseudo-code or English, if it is readable enough.)

Introduction to Algorithms and Data Structures

1. Foundation of Algorithms (1) Basic Programming Models

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What’s an algorithm?

Algorithm = Description of a method of solving a problem using computer

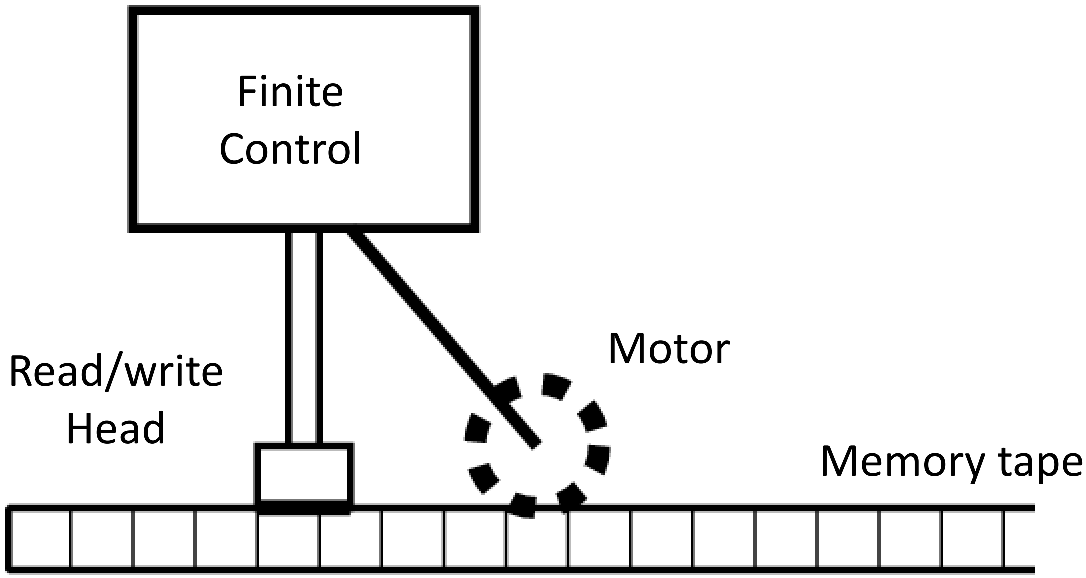
- What’s a good algorithm?
  - It outputs a correct answer for any input
  - It outputs an answer in reasonable cost
    - polynomial time of input length
    - polynomial space of input length
- What’s a bad algorithm?
  - It takes a loooong time for some input
  - It uses a huuuge memory for some input
  - (There exists unsolvable problems by any program)

Models of “computation”

How can we evaluate time and space?
→ First of all, how do computers work?

- Efficiency of algorithms may change according to computation model
  - What are “basic operations”?
  - What kind of data can it store?
    - Natural numbers, real numbers (?), images, musics…?
- There are some standard models
  - Turing machine: That Turing innovated. Base of all computation models.
  - RAM model: Standard model for algorithm theory.
  - We may use models based on GPU and/or quantum computation in near future…

Turing Machine Model

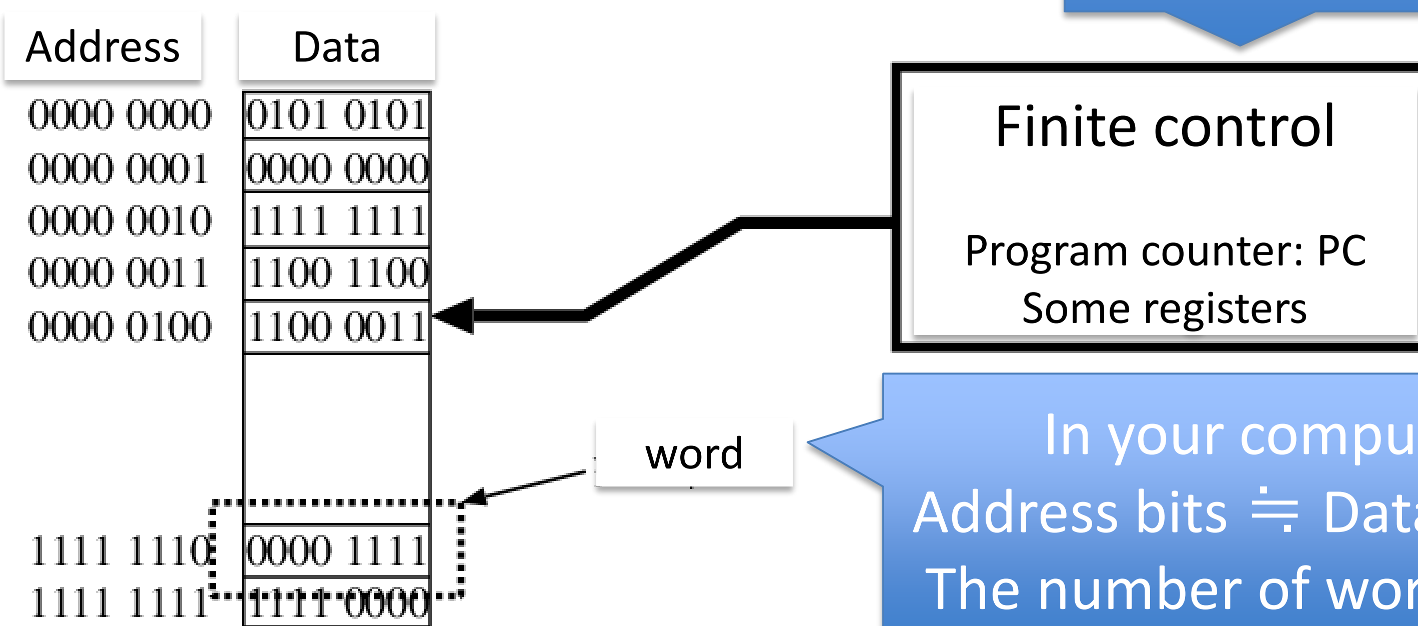


- Simple theoretical model
- Any computable problem is also solvable by a Turing machine
- It is so simple that programming is very tedious
  - No mathematical operations including +, -, ×, ÷
  - It is hard to consider “essence” of algorithms

RAM Model

(Random Access Mem

When we design an algorithm, we suppose memory is so huge that we have no overflow.



In your computer;
Address bits ≒ Data bits = k
The number of words ≦ 2^k

- It consists Memory and CPU (Central Processing Unit)
  - We do not mind Input/Output
- It is essentially the same as your computer
- CPU can access any address randomly (not sequentially) in a unit cycle
- Programming language C is a system that shows you this structure implicitly (like arrays and pointers)

Programming Language

- Compiler translates any “readable” program (for human) to an executable file in machine language (for the CPU)
- E.g. Programming language C; It is okay if you know…
  1. variable
  2. array
  3. pointer
  4. control statement (e.g., if, while)
  5. recursive call

Basic of C: Hello World

- We use C language, but the other languages (C++, C#, Java, etc.) are basically similar
- We give very rough basic programming
- Output “Hello World” on display

```
#include <stdio.h> /*use printf*/

main(){
  printf(“Hello World”);
}
```

statement

Semi-colon after a statement

★ In C#, use System.Out.WriteLine instead of printf.

Basic of C: Arithmetic operations

- Basic operations: `+, -, *, /, %`

Exp.	Meaning
3+4	Add 3 and 4
3-1	Subtract 1 from 3
3*3	Multiply 3 and 3
4/2	Divide 4 by 2
3%2	Reminder by dividing 3 by 2

  - Except `%, the operations can be used for integers (int`, etc.) and real numbers (`float`, `double`, etc.)

Basic of C: Notes for arithmetic ops.

- (int/int) is rounded (by cutting off)
  - Ex: `1/3 is 0, whereas 1.0/3 is 0.3333…`
- `double av = (int)sum/(int)num (Fail)`
- No comma for delimiter
  - Ex: `10,000 is not okay. Write as 10000.`
- We use () to control ordering:
  - We cannot use {} or []
  - Ex: `{(3+4)*3+4}*6` is not correct. Write as `((3+4)*3+4)*6`
- No power operation (we can use `**` in some languages)

Basic of C: Variable

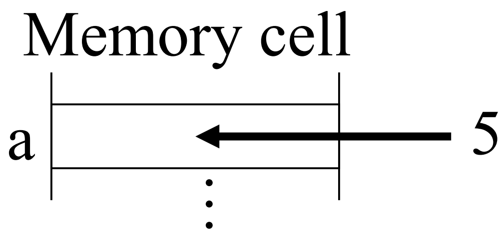
- Variable: It is a memory cell, that indicates the “place” to memory a result of computation
- Rules for naming
  - Start with alphabet (UPPER, lower letters, and _)
  - From the second letter, you can use alphabets and numbers
    - Not any other
  - Upper and lower letters are different
    - FF, ff, fF, and Ff are all different names
  - Not reserved words in C (e.g., main, include, return)

  - Good: `x`, `orz`, `T_T`, `IE9`, `projectX`, `ff4`, `y2k`, `JAIST`
  - Bad: `7th`, `uehara@jaist`, `ac.jp`, `tel#`

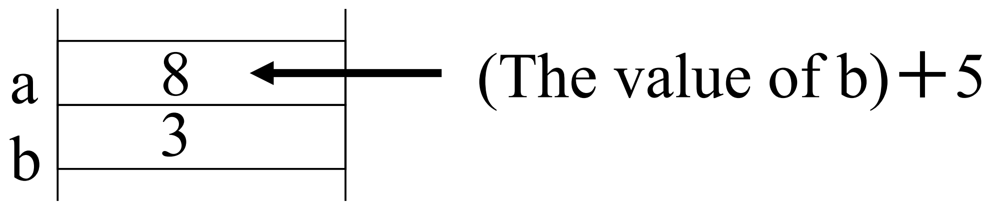
Basic of C: Assignment statement

“=“ is not “equal” in the sense of mathematics

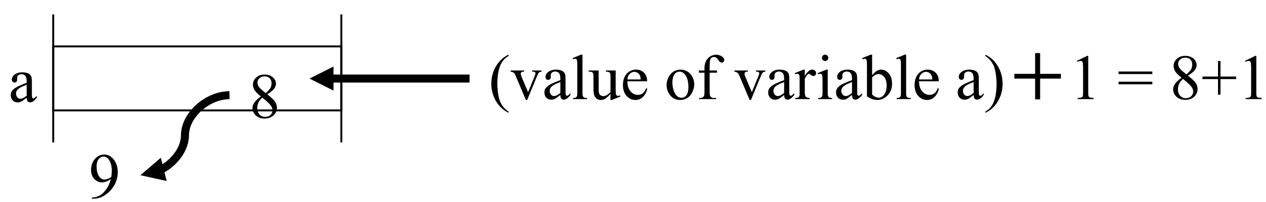
- a=5



  - Store the value 5 to the place named by a in memory
- a=b+5



  - Store value of “value stored at the place named by b (or value of the variable b) plus 5” to the place named by a
- a=a+1



  - Store value of “the value of variable a plus 1” to the place named by a

Basic of C: Declaration of variable

- You have to declare variables beforehand (in C language)

Good

```
main(){
  int a,b;
  a = 5; b = 3;
  printf("a+b=%d",a+b);
}
```

Variables a and b in integer

Bad

```
main(){
  a = 5;
  printf("%d",a);
}
```

It is not good!

Note: Recent language (like python) does not require to declare beforehand. The system guesses and makes simpler, but sometimes causes bugs...

Basic of C: Mathematical functions

	function	Math. symbol	type	Parameter type
Square root	sqrt(x)	$\sqrt{x}$	double	double
Power	pow(x, y)	x^y	double	double
Logarithm	log(x)	$\log_e x$	double	double
Logarithm	log10(x)	$\log_{10} x$	double	double
Exponential	exp(x)	e^x	double	double

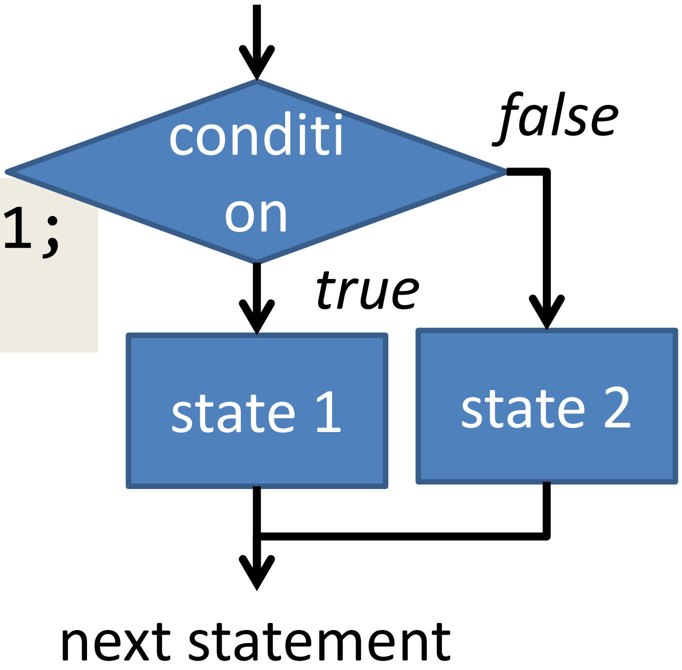
- Source code: include the following header file
  `#include <math.h>`
- Compile: Option -lm is required
  - gcc main.c –lm

★ Write a = Math.sqrt(b) in C#

Basic of C: Control statements
if statement – conditional branch (1/2)

- Grammar

```
if (condition) state 1;
else state 2;
```



If condition is true, perform statement 1, and perform statement 2 if it is false

  – Ex: Output EVEN if n is even, and ODD if it is odd.

```
if(n%2==0) printf("EVEN");
else printf("ODD");
```

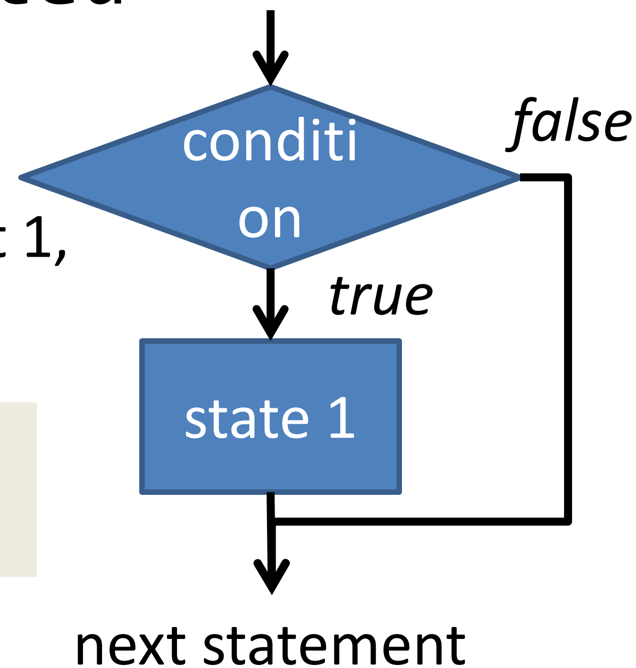
We use "==" to check equality in C.

Basic of C: Control statements if statement – conditional branch (2/2)

- else part can be omitted

```
if(condition) state 1;
```

If condition is true, perform statement 1, and perform nothing if it is false




```
What happens??:
if(condition) state 1; state 2;
```

```
Write as follows:
if(condition) {
  state 1;
  state 2;
}
```

Basic of C: Representations of conditions (1/2)

<table>
<tr><th>symbol</th><th>meaning</th><th>example</th><th>meaning of example</th></tr>
<tr><td>==</td><td>equal</td><td>n == 2</td><td>n is equal to 2</td></tr>
<tr><td>!=</td><td>not equal</td><td>n != 0</td><td>n is not equal to 0</td></tr>
<tr><td>></td><td>greater than</td><td>n > 3</td><td>n is greater than 3</td></tr>
<tr><td>>=</td><td>g.t. or equal</td><td>n >= 3</td><td>n is g.t. or equal to 3</td></tr>
<tr><td><</td><td>less than</td><td>n < 0.01</td><td>n is less than 0.01</td></tr>
<tr><td><=</td><td>l.t. or equal</td><td>n <= 0.01</td><td>n is l.t. or equal to 0.01</td></tr>
<tr><td>&&</td><td>and</td><td>0 < n && n <= 10</td><td>n is greater than 0 and less than or equal to 10</td></tr>
<tr><td>||</td><td>or</td><td>n < 0 || 0 < n</td><td>n is less than 0 or greater than 0</td></tr>
<tr><td>!</td><td>not</td><td>!(n < 0.01)</td><td>n is not less than 0.01</td></tr>
</table>

Basic of C: Representations of conditions (2/2)

- You cannot compare 3 or more items
  - `0<x<5` ➔ 0 < x && x < 5
  - `a==b==c` ➔ a == b && b == c

- Example: Check of leap year
  - Dividable by 400, or
  - Not dividable by 100 but dividable by 4

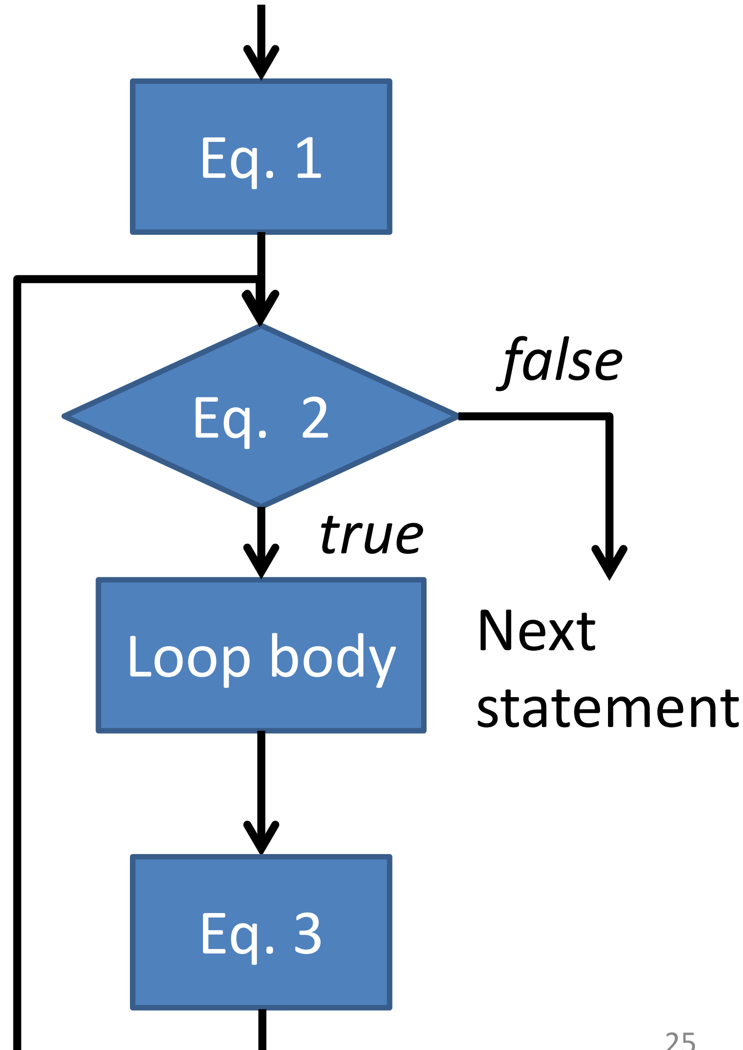
```
year%400==0 || (year%100!=0 && year%4==0)
```

Basic of C: Control statements for loop – repeating (1/4)

- Grammar

```
for(eq.1;eq.2;eq.3){
  loop body
}
```

- It runs as follows:
  A) Execute eq. 1
  B) If eq.2 is *true*, step C, and step D if *false*
  C) Perform loop body and eq. 3, jump to B
  D) Go to next statement



At a glance, it seems to be complex,
but we have several standard patterns.

Basic of C: Control statements
for loop – repeating (2/4)

Example: Output the sum $\sum_{i=1}^{n} i$ from 1 to n

```
int i,n,sum;
n=/*initialized somehow*/;
sum=0;
for(i=1;i<=n;i=i+1){
  sum=sum+i;
}
printf("1+…+%d=%d",n,sum);
```

In C,
you can write i++ instead of i=i+1, and

you can write sum+=i instead of sum=sum+i

★ You may write as System.Console.WriteLine ("1+…+"+n+"="+sum) in C#

Basic of C: Control statements
for loop – repeating (3/4)

Example: Output the sum $\sum_{i=1}^{n} i^2$ from 1 to n

```
int i,n,sum;
n=/*initialized somehow*/;
sum=0;
for(i=1;i<=n;i=i+1){
  sum=sum+i*i;
}
```

Basic of C: Control statements for loop – repeating (4/4)

- Ex: Compute $\sum_{i=1}^{n}(2i-1)^2$

```
int i,n,sum;
n=/*initialized somehow*/;
sum=0;
for(i=1;i<=2n-1;i=i+2){
  sum=sum+i*i;
}
```

i indicates 2j-1

- Why is this correct?
  - Because; $\sum_{i=1}^{n}(2i-1)^2 = 1^2 + 3^2 + \cdots + (2n-1)^2$

Basic of C: Control statements for loop – repeating (4/4) suppl.

- Ex: Compute $\sum_{i=1}^{n} (2i-1)^2$

```
int i,n,sum;
n=/*initialized somehow*/;
sum=0;
for(i=1;i<=n;i=i+1){
  sum=sum+(2*i-1)*(2*i-1);
}
```

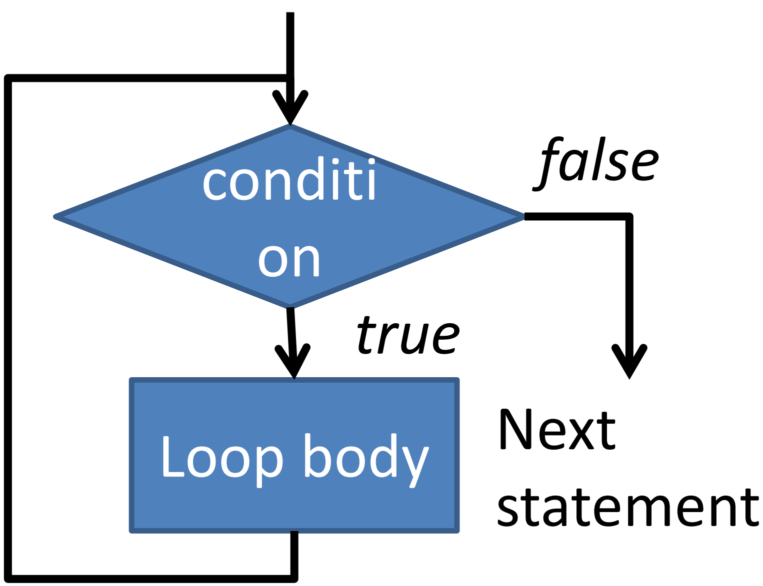
- Of course, you can do in this way.

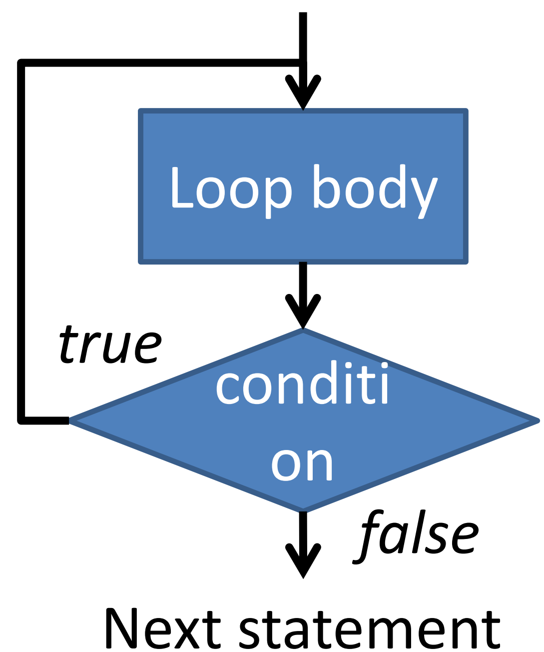
Basic of C: Control statements while loop & do-while loop (1/2)

- Grammar

```
while(condition){
  loop body
}
```

```
do{
  loop body
}while(condition)
```





Basic of C: Control statements while loop & do-while loop (2/2)

Ex: Compute GCD(a,b) of two integers a and b

```
int a,b,r;
a=/*some value*/;
b=/*some value*/;
do{
  r = a % b;
  a = b; b = r;
}while(r!=0);
printf("G.C.D.=%d",a);
```

Ex: a=1848, b=630

a	b	r=a%b
1848	630	588
630	588	42
588	42	0
42	0	0

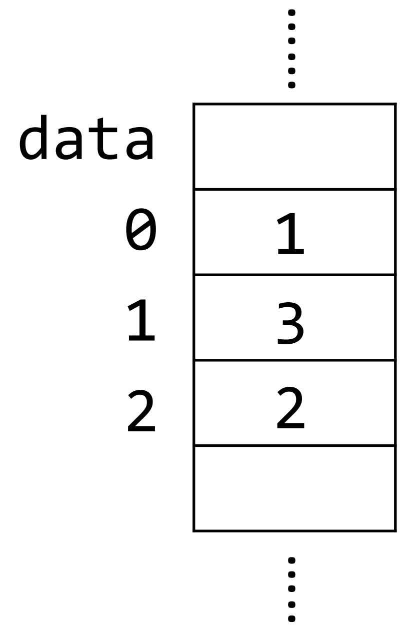
This method (algorithm) is known as "Euclidean mutual division method", which is known as the oldest algorithm.

★ In C#, int[] data = new int[3];

Basic of C: Array (1/2)

Not only "values" in recent language.

- What is array?
  Data structure that aligns many data in the same type (`int`, `float`, etc.) sequential in memory
- Ex: `int data[3]`
  - 3 consecutive memory cells are kept as name "data", in which each cell stores an integer.




```
int data[3];
data[0]=1;
data[2]=2;
data[1]=3;
```

Basic of C: Array (2/2)
Get the maximum

- Ex: compute the maximum value in integer data[100]

Wrong!

```
int data[100];
int i,max;
/*data is initialized somehow*/
max=0;
for(i=0;i<100;i=i+1){
  if(max<data[i]) max=data[i];
}
printf("maximum data = %d",max);
```

Q: Is this program correct?

Basic of C: Array (2/2) Get the maximum

- Ex: compute the maximum value in integer data[100]

```
int data[100];
int i,max;
/*data is initialized somehow*/
max=0;
for(i=0;i<100;i=i+1){
  if(max<data[i]) max=data[i];
}
printf("maximum data = %d",max);
```

Wrong!

When all data is negative, it outputs 0 as the maximum!

Q: Is this program correct?

Basic of C: Array (2/2)
Get the maximum

- Ex: compute the maximum value in integer data[100] – make it correct

```
int data[100];
int i,max;
/*data is initialized somehow*/
max=data[0];
for(i=1;i<100;i=i+1){
  if(max<data[i]) max=data[i];
}
printf("maximum data = %d",max);
```

The value of max is always in data

10 minutes report

- What does the following function compute?
  - Find the outputs of collatz(5) and collatz(7)

```
collatz(unsigned int n) {
  print(n);  // output n
  if (n == 1) return;
  if (n%2==0) collatz(n/2);
  else        collatz(3n+1);
}
```

Function calls itself recursively with different parameters

1 day report (1/2)

- Definition of ExOR $\oplus$:
  - $0 \oplus 0=0$, $0 \oplus 1=1$, $1 \oplus 0=1$, $1 \oplus 1=0$

"Exclusive OR" operation

- For integers in binary system, we apply ExOR bitwise; for example,
  - $10_{10} \oplus 7_{10} = 1010_2 \oplus 111_2 = 1101_2 = 13_{10}$

1. Compute the following
   1. $8_{10} \oplus 3_{10}$
   2. $15_{10} \oplus 7_{10}$

1 day report (2/2)

2. What does this function S(x,y) do?

```
S(int x, y) {
   x=x ⊕ y;
   y=x ⊕ y;
   x=x ⊕ y;
}
```

Hint: Try computing
(x=8, y=3),
(x=15, y=7),
(x=1, y=128),
and so on…

Submit the report on
Wednesday, 10:00am.