Introduction to Algorithms and Data Structures

Lesson 17: Super Application *Computational Origami*

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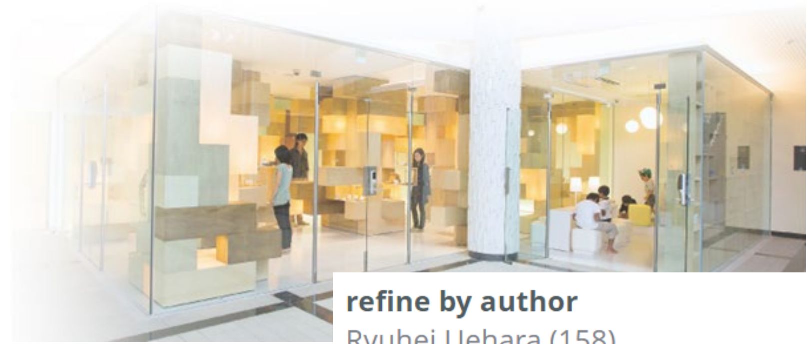

Self introduction

Affiliation:

JAIST School of Information Science

Professor

DBLP Info.:

refine by author
Ryuhei Uehara (158)
Erik D. Demaine (39)
Takeaki Uno (27)
Yota Otachi (27)
Yushi Uno (26)
Martin L. Demaine (22)
Toshiki Saitoh (19)
Takehiro Ito (17)
Yoshio Okamoto (16)
Takashi Horiyama (13)
127 more options

refine by venue
CCCG (18)
ISAAC (14)
WALCOM (12)
Theor. Comput. Sci. (12)
CoRR (11)
IEICE Transactions (9)
TAMC (7)
Bulletin of the EATCS (6)
FUN (4)
Discrete Applied Mathematics (4)
37 more options

Specialist of Theoretical Computer Science

- Algorithms
  - Graph Algorithms
- Computational Complexity
  - of Puzzles and Games…
  - Recreational Mathematics
- Computational Geometry
  - Computational Origami

Erdös number＝2 (with Pavol Hell)

Director of JAIST Gallery (with more than 10000 puzzles)

I'd like to give some talks in the last day…?

Introduction to Computational Origami

CANDAR Keynote 2:
Folding and Unfolding Algorithms on (Super)Computer

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Computational ORIGAMI

- "ORIGAMI"
  - In 1500s, may be in Asia, with "papers"…?
  - Now "ORIGAMI" is popular even in English; There are many Origami books in book stores.
  - Something like "Origami"… while "Ori" means *folding*, and "gami" means *paper*…

There are many origami-applications or origami-engineering even they are not "folding", not "paper"…; e.g., DNA folding, folding robots, …

Computational ORIGAMI

- Development of recent Origami
  - In 1980s – 1990s, Origami becomes complicated, which is called “complex origami”.

Maekaya Devil, 1980. (From one square sheet of paper)

Kawasaki Rose, 1985. (From one square sheet of paper)

Cuckoo Clock by Robert Lang, 1987. (From one rectangular sheet of size 1x10)

Computational ORIGAMI

- Computerized Origami…
  - Since 1990s, computer aided design of origami popular.

In 2016, they were key items in movies “Shin-Godzilla” and “Death Note”

Cuckoo Clock by Robert Lang, 1987. (From one rectangular sheet of size 1x10)

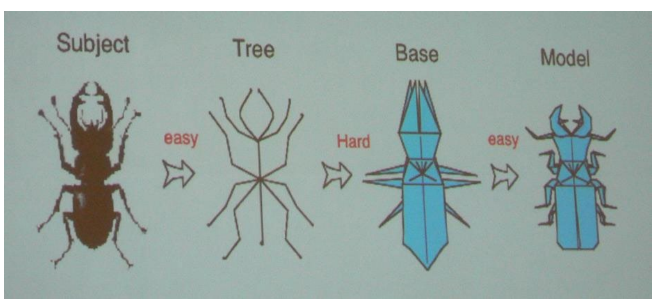
Origamizer by Tomohiro Tachi, 2007. (From one rectangular sheet in 10 hours ;-)

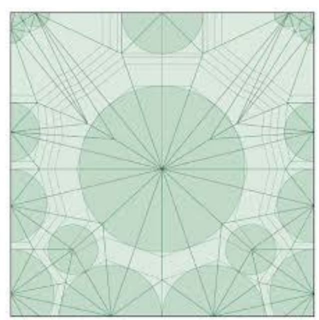
Mathematically designed origami Jun Mitani, 2010. (From one rectangular sheet

Origami and Computer Science

- Development of Design method with computer
  - 1980s：Maekawa’s Devil
    - Get “parts” together in a CAD-like way
    - So called “Complex Origami” has been developed
  - 2000s：“TreeMaker”; software by Robert Lang
    - Any given “metric tree” is developed into a square sheet of paper such that folding the crease pattern, you can get “large” metric tree.
    - Practical algorithm that solves several optimization problems.

Including NP-hard problems





International Conferences on Origami

1. December, 1989@ Italy
   The International meeting of Origami Science and Technology
2. 1994@Shiga, Japan
3. March, 2001@USA
   The International meeting of Origami Science, Mathematics, and Education (3OSME)
4. August, 2006@USA
   4OSME
5. July, 2010@Singapore
   5OSME
6. August, 2014@Tokyo, Japan
   6OSME
7. September, 2018: 7OSME@Oxford, UK.

Origami and Computer Science

- Proposal of "Computational Origami"

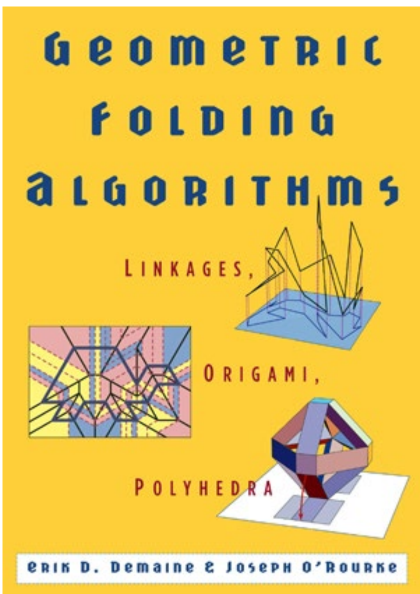
  Since 1990s, in Computational Geometry Society, "folding problems" are investigated in the contexts of "computational geometry" and "optimization problems"

  Very famous researcher in this area: Erik D. Demaine

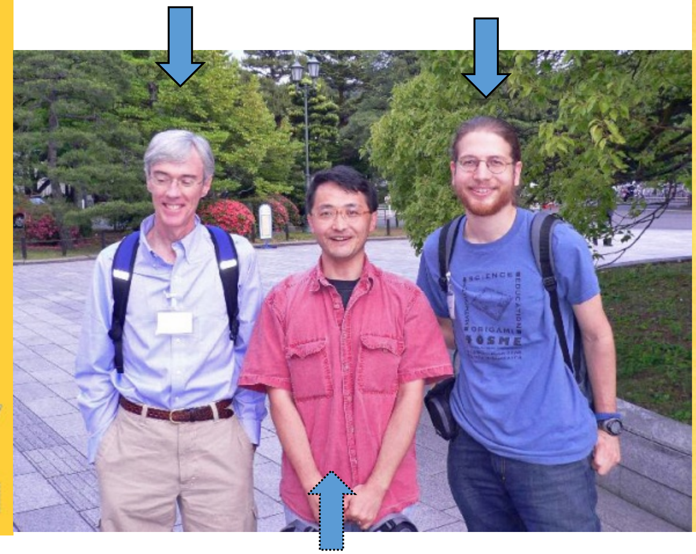
  - He was born in 1981
  - In 2001, he got Ph.D when he was 20, and became faculty member in MIT
  - Topic of his Ph.D thesis was computational origami
  - Still leading Origami research at MIT! (e.g., origami-robots)

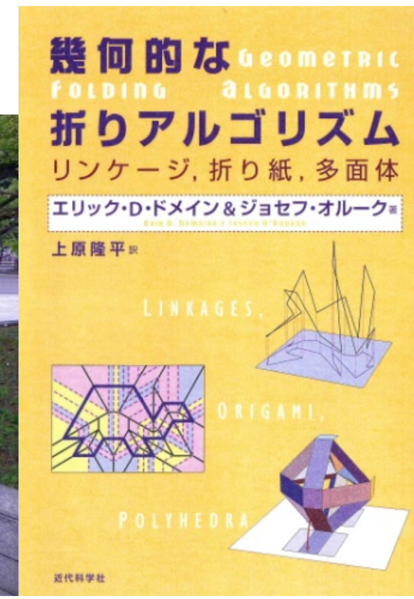
Origami and Computer Science

- “Bible” in Computational Origami
  J. O’Rourke and E. D. Demaine, *Geometric Folding Algorithms: Linkages, Origami, Polyhedra*, 2007.



Authors





I translated into Japanese (2009).

Today's Topic

Relationship between polygon and convex polyhedron folded from it

- Big open problem and related problems
- For a given polygon, how can we compute (convex) polyhedron folded from it?
  - This problem is related to both of
    - Computational geometry
    - Graph theory and graph algorithms
  - We need "mathematical property", "nice algorithms", and "computer power"!

Today's Problem: Folding 2 or more boxes from one polyomino (polygon made by unit squares)

There are many open problems, and young researchers had been solving them ☺

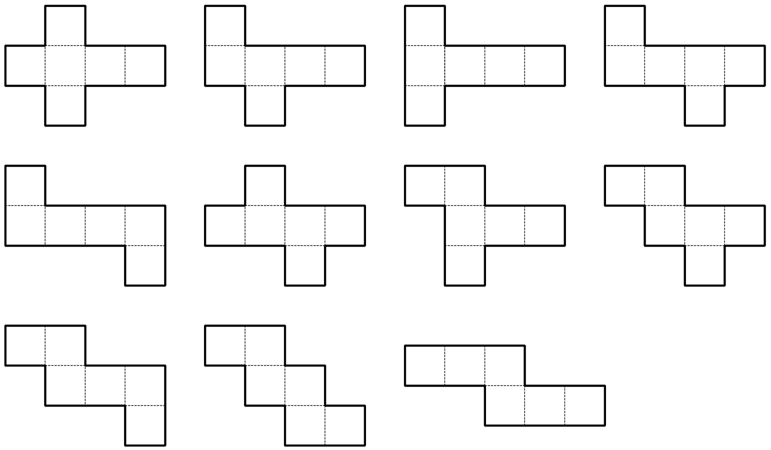
Prelim: (Edge) unfolding

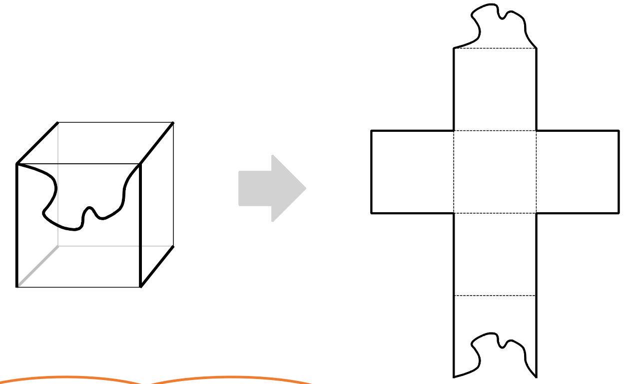
- **(General) development**: polygon obtained by cutting any surface of a polyhedron and developing of it.
  - It should be connected.
  - It should be non-overlapping simple polygon.
- **(Edge) development**: development by cutting along edges of the polyhedron
  - Boundary of development consists of edges of polyhedron
  - In Japanese elementary school, we had learnt this notion as “development”, which I don’t know why?

★Today’s ”Development” means general ones!

Exercise: Unfolding Puzzle!

- We learnt "a cube has 11 different developments" in elementary school. But it is not in our context; there are infinitely many.
- Puzzle: Find the other developments that consist of 6 squares.
  1. They can be different sizes!
  2. Can you find ones that consists of 6 unit squares?





Special Thanks:
Masaka Iwai

If you know traditional origami "Balloon",,, ☺

Prelim. Basic facts

Let G be a graph induced by the vertices and edges of a convex polyhedron S:

[Theorem 1]
Cut lines of any edge development of S produces a spanning tree of G

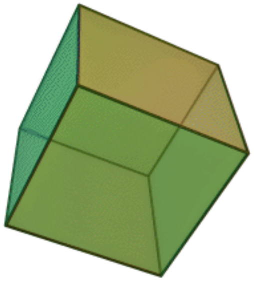
[Proof]

- It visits all vertices: If not, uncut vertex cannot be flat.
- It produces no cycle:
  If not, the development cannot be connected.

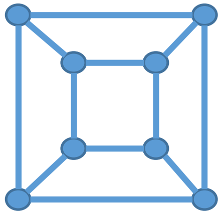
[Theorem 2]
Cut lines of any general development of S a tree that spans all vertices of S.

Note: We say nothing about overlapping, which is the other (and quite difficult) problem.

S

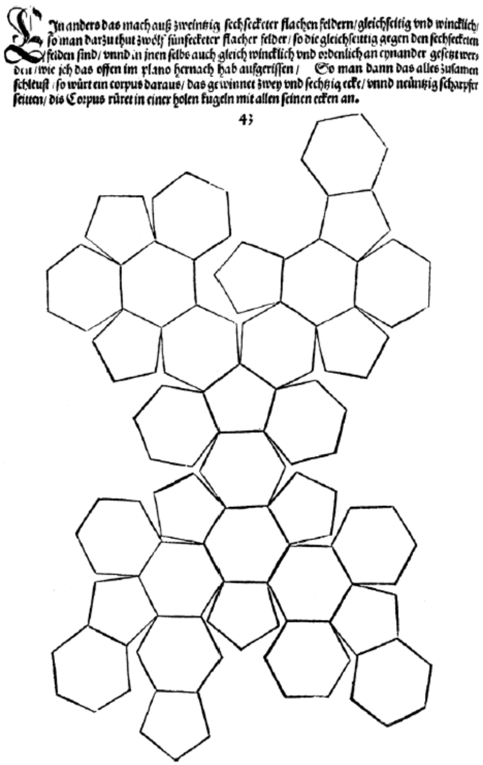


G



Quick History

- In *Underweysung der Messung* (Albrecht Dürer, 1525), Dürer described many solids by their developments;



Ein anders das mach auß zweintzig sechseckter flachen feldern/ gleichseitig vnd wincklich/ so man darzu thut zwölff fünfeckter flacher felder/ so die gleichseittig gegen den sechseckten felden sind/ vnnd in jnen selbs auch gleich wincklich vnd ordenlich an eynander gesetzt werden/ wie jch das offen im plano hernach hab aufgerissen/ So man dann das alles zusamen schleust/ so würt ein corpus daraus/ das gewinnet zwey vnd sechtzig ecke/ vnnd neüntzig scharpffe seitten/ dis Corpus rüret in einer holen kugeln mit allen seinen ecken an.

43

He conjectured the following?

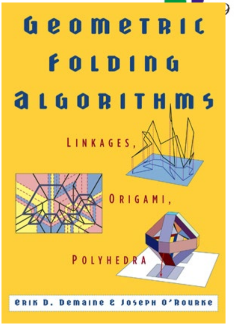
Big open problem:

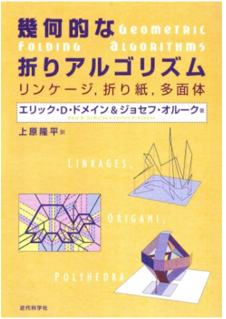
Any convex polyhedron has an edge development, i.e.,

- Connected
- Non-overlapping

Quick History

See the book if you are interested in this topic…





Open problem:

Any convex polyhedron has an edge unfolding.

Related results (I don’t talk anymore today);

- Counterexample when you consider non-convex ones
  (any edge development causes overlapping)
- We have algorithms if you allow general unfolding
  (cut along all shortest paths from one point to all vertices)
- Experimentally, random edge unfolding of a random convex polyhedron causes overlapping with probability almost 1.

Summary : <u>We have few knowledge about development</u>

Target of this research:

- Given a polygon P, determine convex polyhedra Q that can be folded from P, and vice versa. (mathematical/computational/…)

Common developments of boxes

- Common developments that can fold to 2 different boxes.
- Common developments that can fold to 3 different boxes…

… and open problems

You can find articles in a monthly magazine in Japan…

My result is used in main trick in a mystery (?) novel!

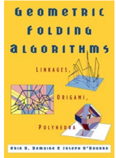
Common developments of boxes

References:

- Dawei Xu, Takashi Horiyama, Toshihiro Shirakawa, Ryuhei Uehara: Common Developments of Three Incongruent Boxes of Area 30, *COMPUTATIONAL GEOMETRY: Theory and Applications*, Vol. 64, pp. 1-17, August 2017.
- Toshihiro Shirakawa and Ryuhei Uehara: Common Developments of Three Incongruent Orthogonal Boxes, *International Journal of Computational Geometry and Applications*, Vol. 23, No. 1, pp. 65-71, 2013.
- Zachary Abel, Erik Demaine, Martin Demaine, Hiroaki Matsui, Guenter Rote and Ryuhei Uehara: Common Developments of Several Different Orthogonal Boxes, *Canadian Conference on Computational Geometry* (CCCG' 11), pp. 77-82, 2011/8/10-12, Toronto, Canada.
- Jun Mitani and Ryuhei Uehara: Polygons Folding to Plural Incongruent Orthogonal Boxes, *Canadian Conference on Computational Geometry* (CCCG 2008), pp. 39-42, 2008/8/13.

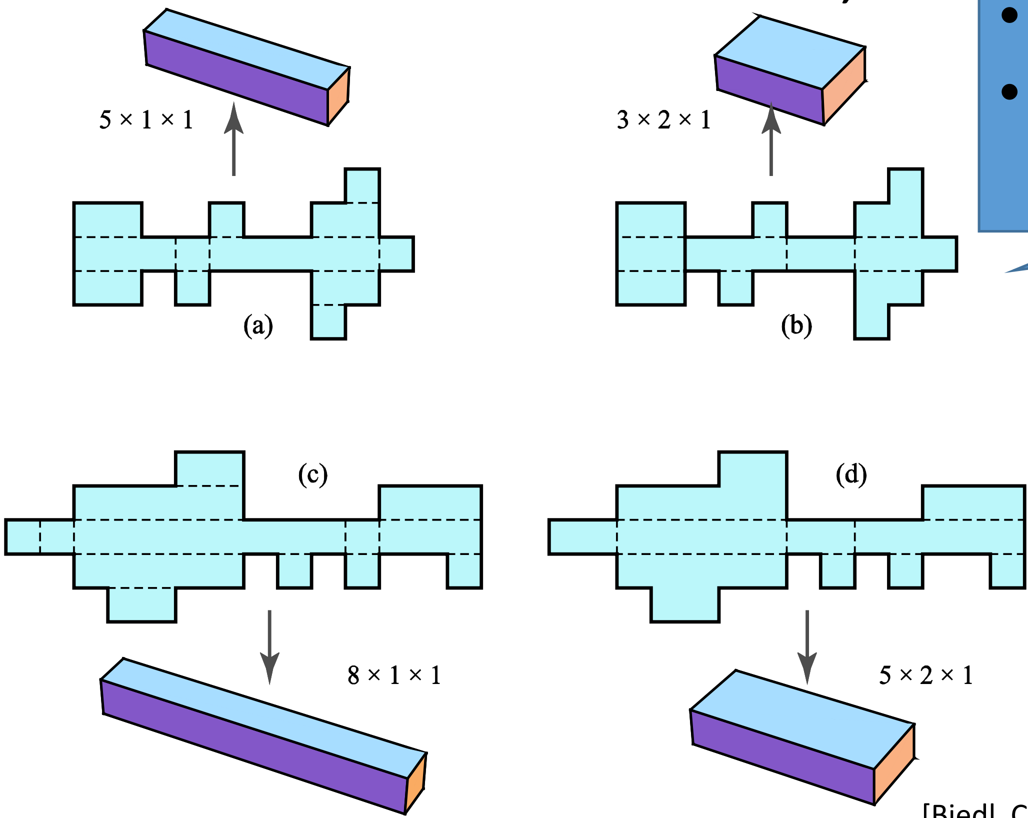
…and some developments:

http://www.jaist.ac.jp/~uehara/etc/origami/nets/index-e.html

When I was translating  …

There are two polygons that can fold to

two different boxes;



- Are they “exceptional?”
- Polygons that fold to 3 or more boxes?

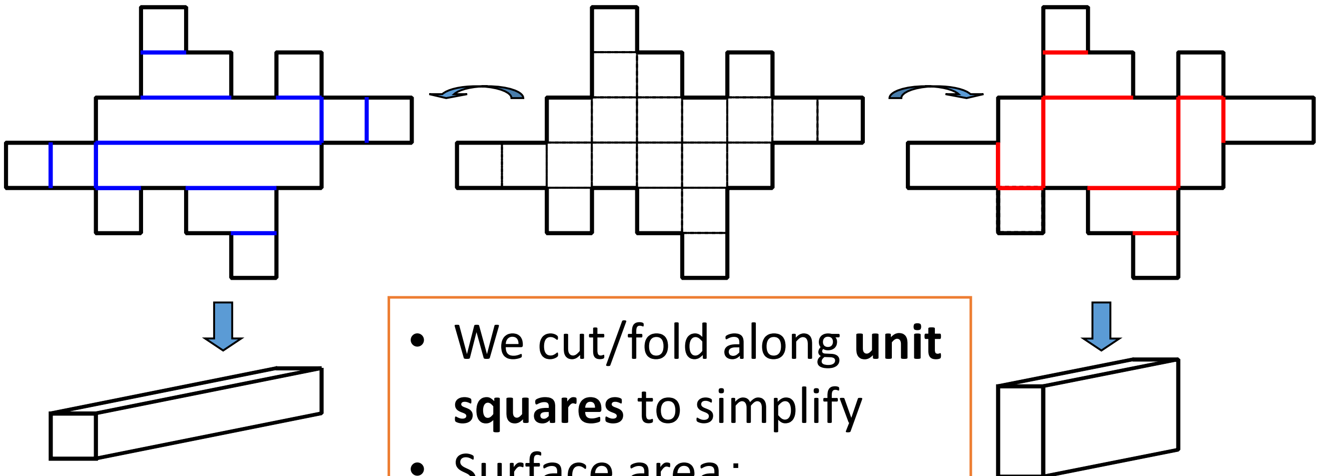
Biedl : I guess you cannot fold 3 boxes by one polygon…

[Biedl, Chan, Demaine, Demaine, Lubiw, Munro, Shallit, 1999]

Before computation…

Example
1×1+1×5+1×5=1×2+2×3+1×3=11 (Area: 22)

When a polygon can fold to 2 different boxes,



$1\times1\times5 = a \times b \times c$

- We cut/fold along **unit squares** to simplify
- Surface area :
  $$2(ab+bc+ca)$$
- Necessary condition:

$$ab+bc+ca = a'b'+b'c'+c'a'$$

$1\times2\times3 = a' \times b' \times c'$

Good areas have many 3-tuples

Precomputation:
Surface areas and possible size of boxes

If you want to find common developments of three boxes,

If you want to find common developments of four boxes,

Area	3-tuples	Area	3-tuples
<u>22</u>	(1,1,5),(1,2,3)	46	(1,1,11),(1,2,7),(1,3,5)
30	(1,1,7),(1,3,3)	70	(1,1,17),(1,2,11),(1,3,8),(1,5,5)
<u>34</u>	(1,1,8),(1,2,5)	94	(1,1,23),(1,2,15),(1,3,11),(1,5,7),(3,4,5)
38	(1,1,9),(1,3,4)	118	(1,1,29),(1,2,19),(1,3,14),(1,4,11),(1,5,9),(2,5,7)

Known results

Polygons that fold to 2 boxes

In [Uehara, Mitani 2008], I ran a randomized algorithm that unfolds many target boxes of several sizes (infinitely :-)

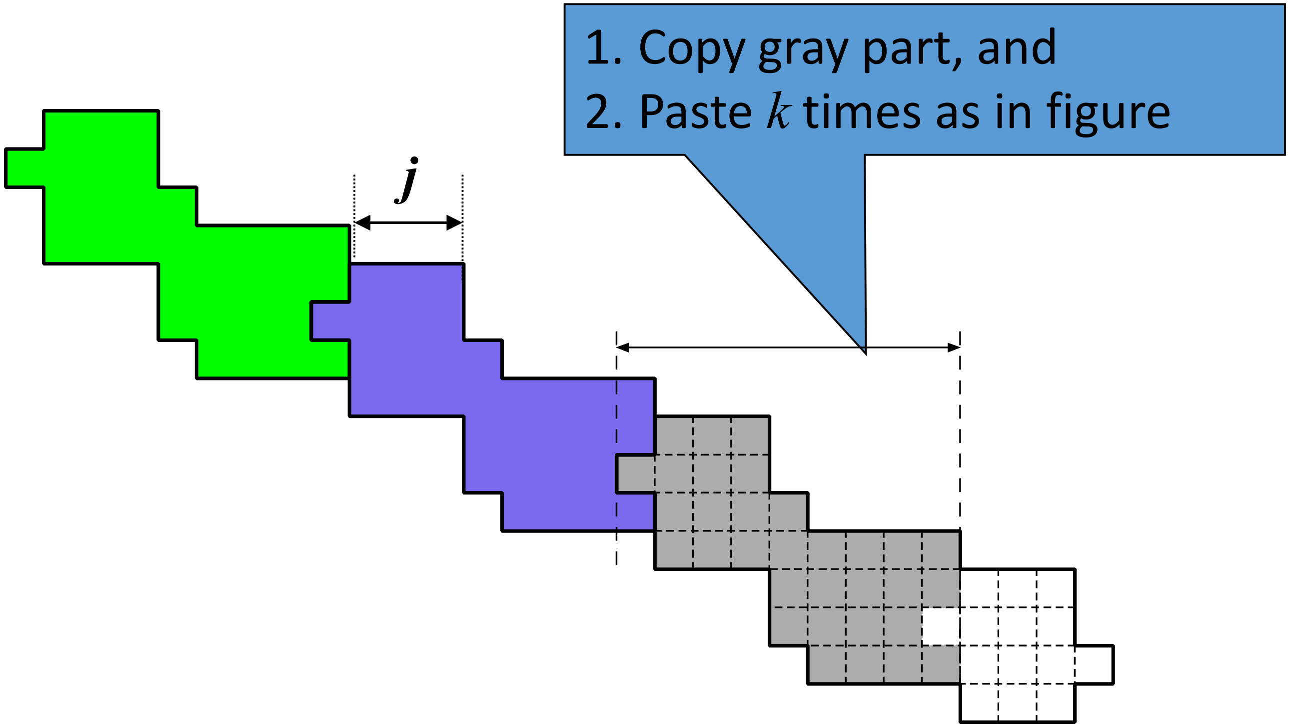
- That fold to 2 boxes;
  1. There are pretty many (～9000)
     (by Supercomputer SGI Altix 4700)
  2. Theoretically,
     there are infinitely many!

- To 3 boxes…?

Common developments of 2 boxes

[Theorem] There are infinitely many common developments of 2 boxes.

[Proof]



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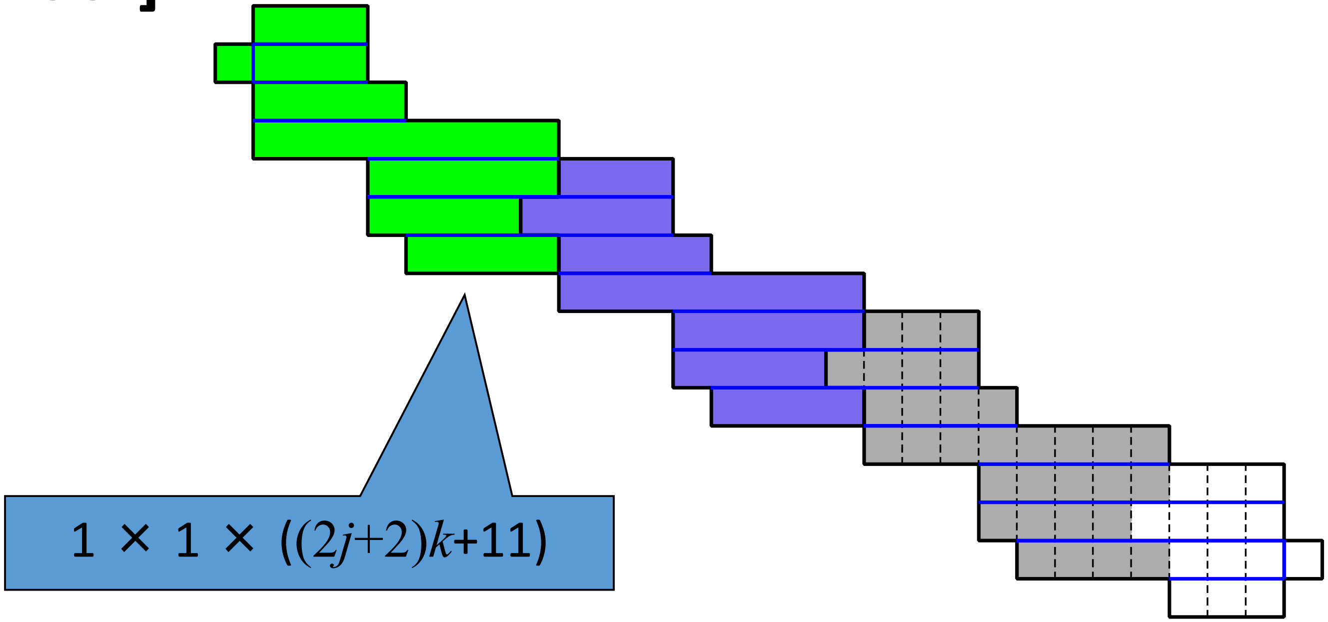

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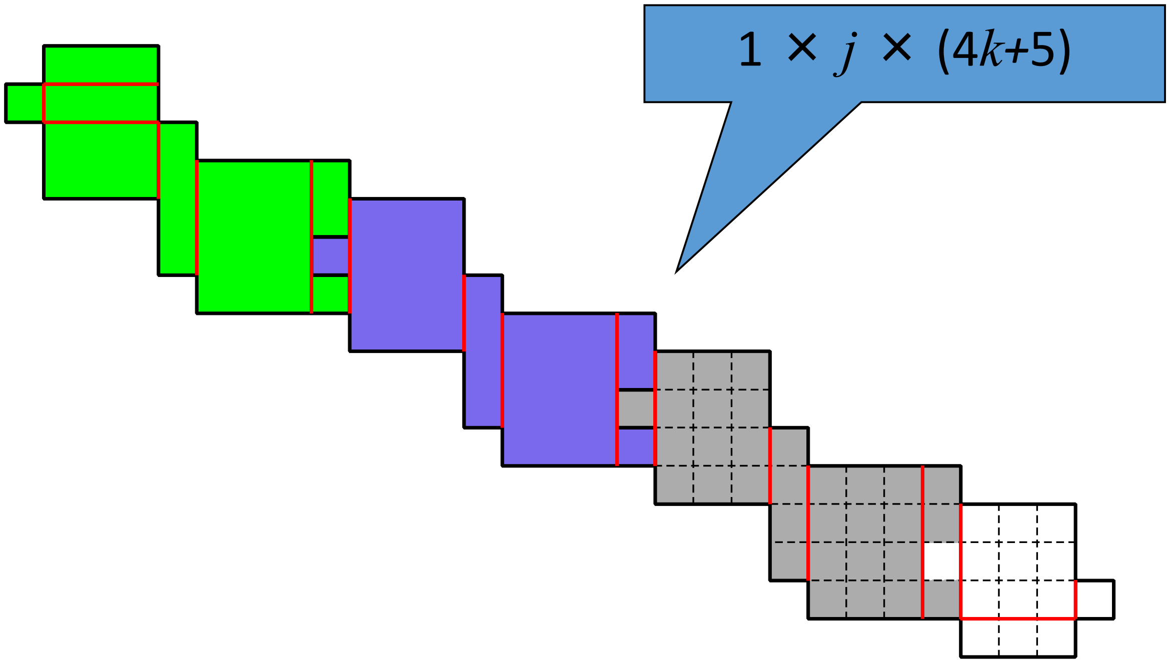
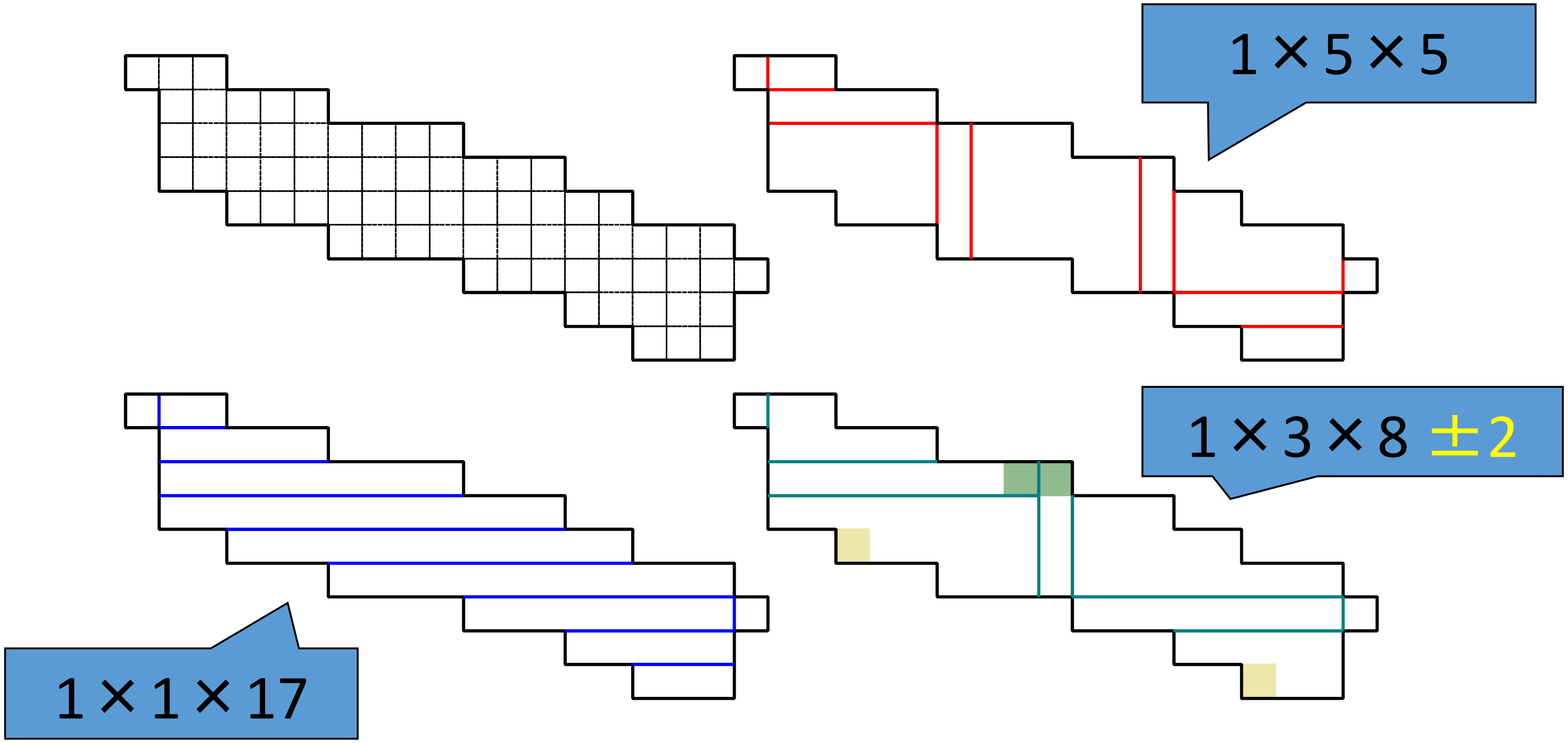


Common development of 3 boxes?

Is there a common development of 3 boxes?

- Pretty close solution among 2 box solutions of area 46:



Challenge to common development of 3 boxes

In [Abel, Demaine, Demaine, Matsui, Rote, Uehara 2011]

- The number of common developments of area 22 that fold into two boxes of size 1×1×5 and 1×2×3 is 2263 in total.
  - Program in 2011: It ran around 10 hours on a desktop PC.
- Among these 2263 common developments, there is only one pear development…

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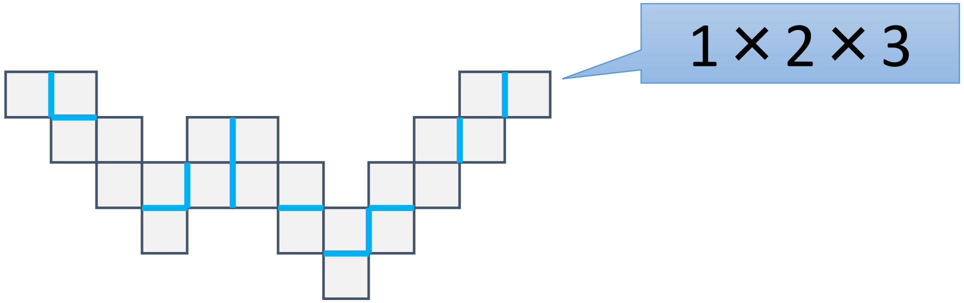

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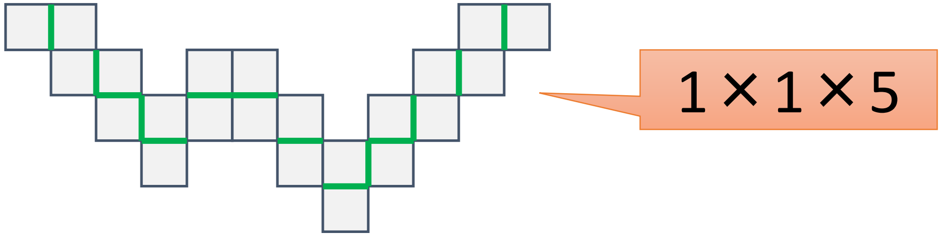

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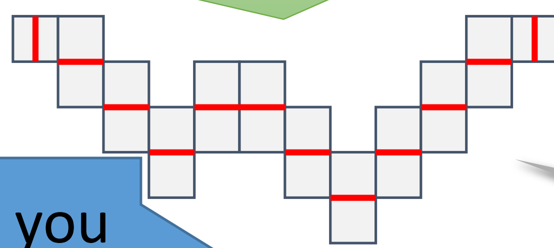
Is it cheating using "box" of volume 0?

Each column has 2 squares, so we can fold it vertically



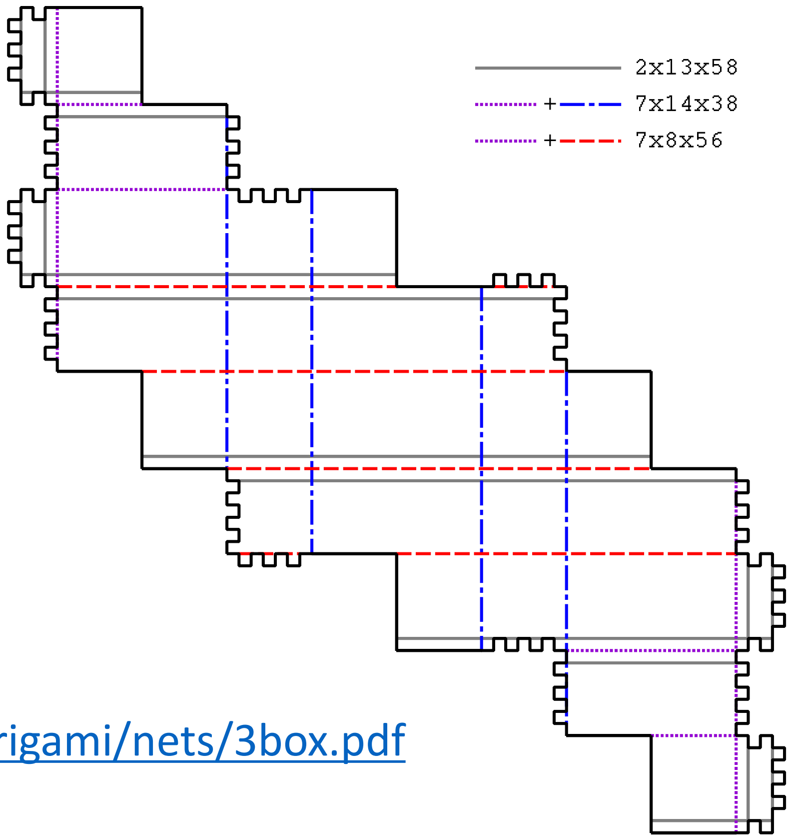
1×11×0 !

If you don't like 1/2 , you can refine each square (□) into 4 squares (田)

Finally: Common developmet of 3 boxes (1)

- February 2012, Shirakawa and Uehara finally found a common development of 3 boxes!!

[Basic idea] We fold one more box from a common development of 2 boxes in somehow….



You can find this pattern at
http://www.jaist.ac.jp/~uehara/etc/origami/nets/3box.pdf

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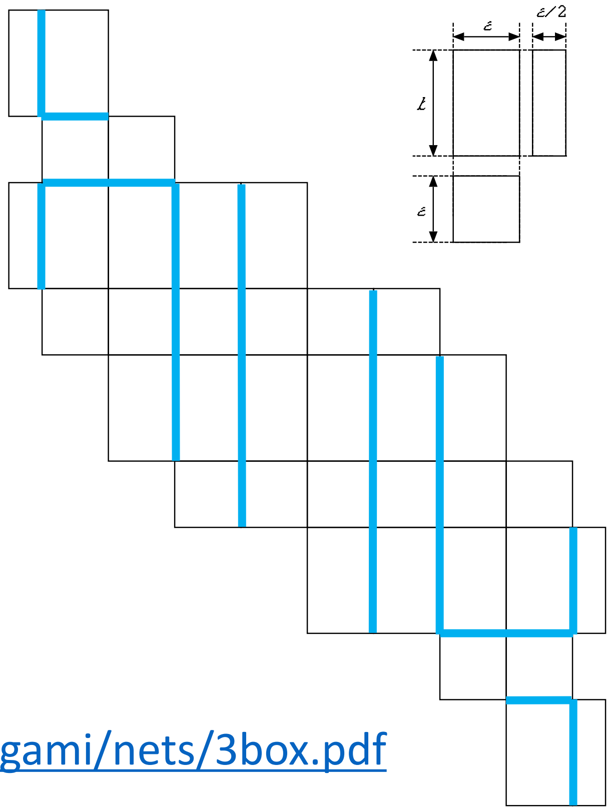

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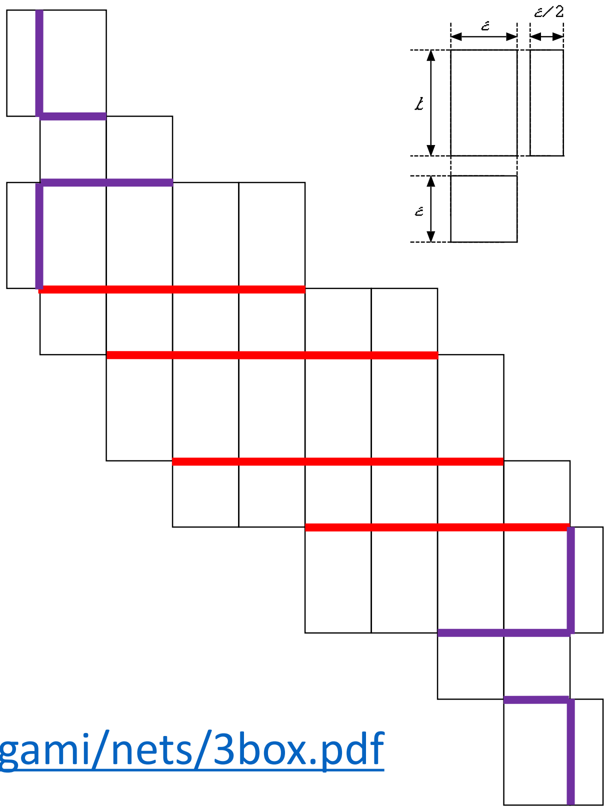

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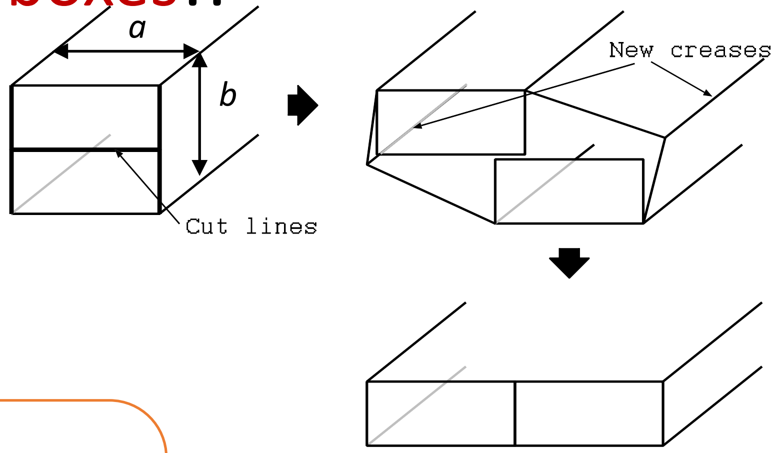

[No!!]
The idea works only when $a=2b$, which allow to translate from a rectangle of size 1×2 to a rectangle of size 2×1.

We may *squash* the box like this way?

http://www.jaist.ac.jp/~uehara/etc/origami/nets/3box.pdf

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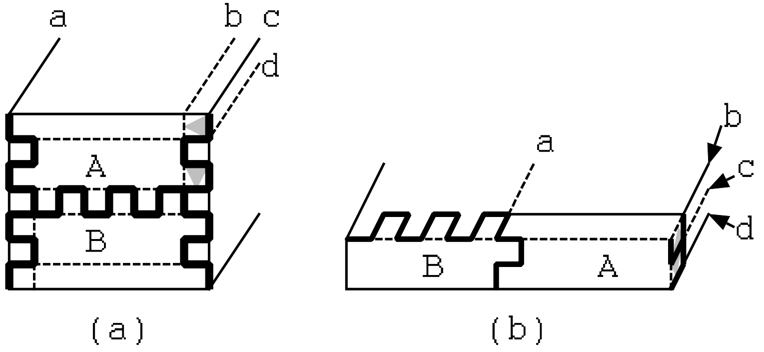

[Yes!!]
If we use a neat pattern!

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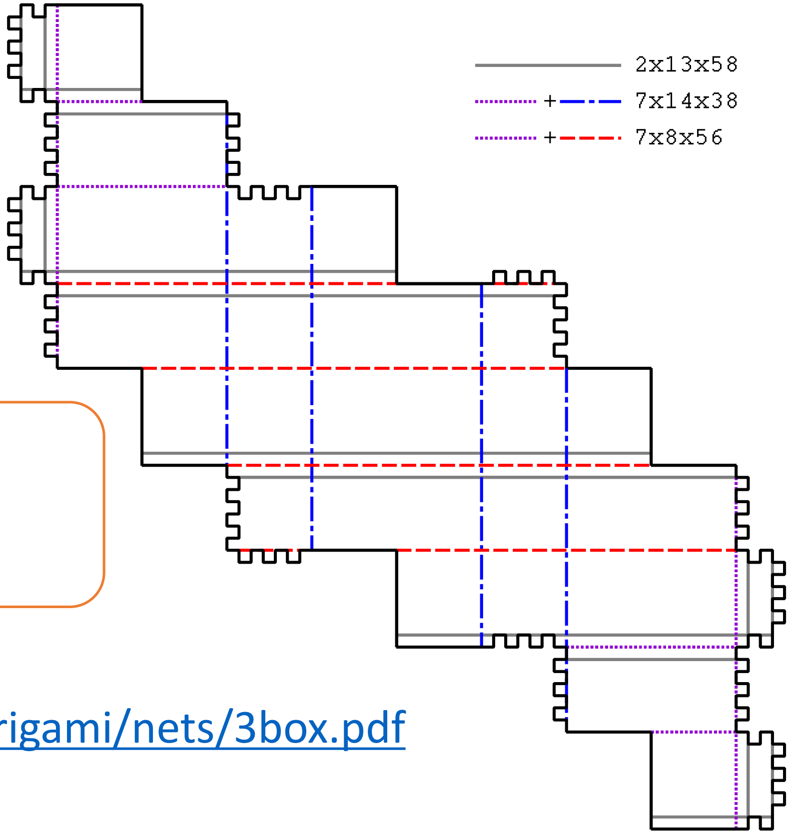

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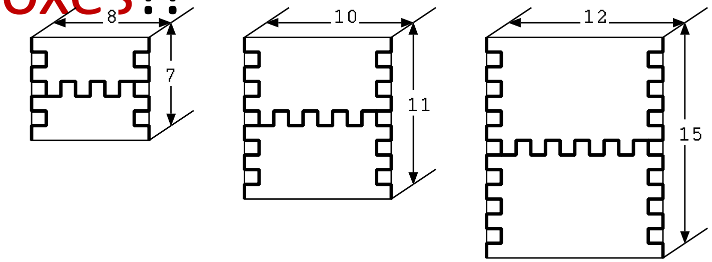


[Basic idea] We fold one more box from a common development of 2 boxes in somehow….

[Theorem]
There are infinitely many polygons that fold to three different boxes.

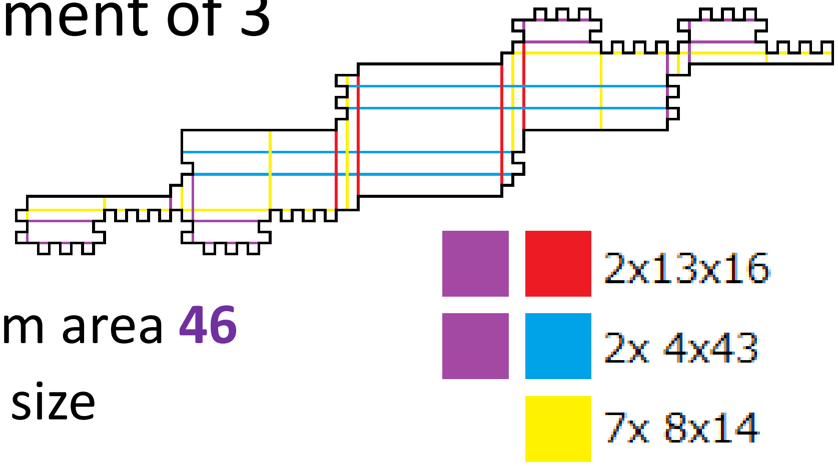
[Generalization]
- The base box has edges of flexible lengths.
- Zig-zag pattern can be generalized.

You can find this pattern at
http://www.jaist.ac.jp/~uehara/etc/origami/nets/3box.pdf

Future work in those days

- The smallest common development of 3 boxes?

Using the idea, we obtain smallest one with **532 unit squares,** which is quite larger than the minimum area **46** that **may** allow us to fold 3 boxes of size $1\times 1\times 11$, $1\times 2\times 7$, $1\times 3\times 5$.

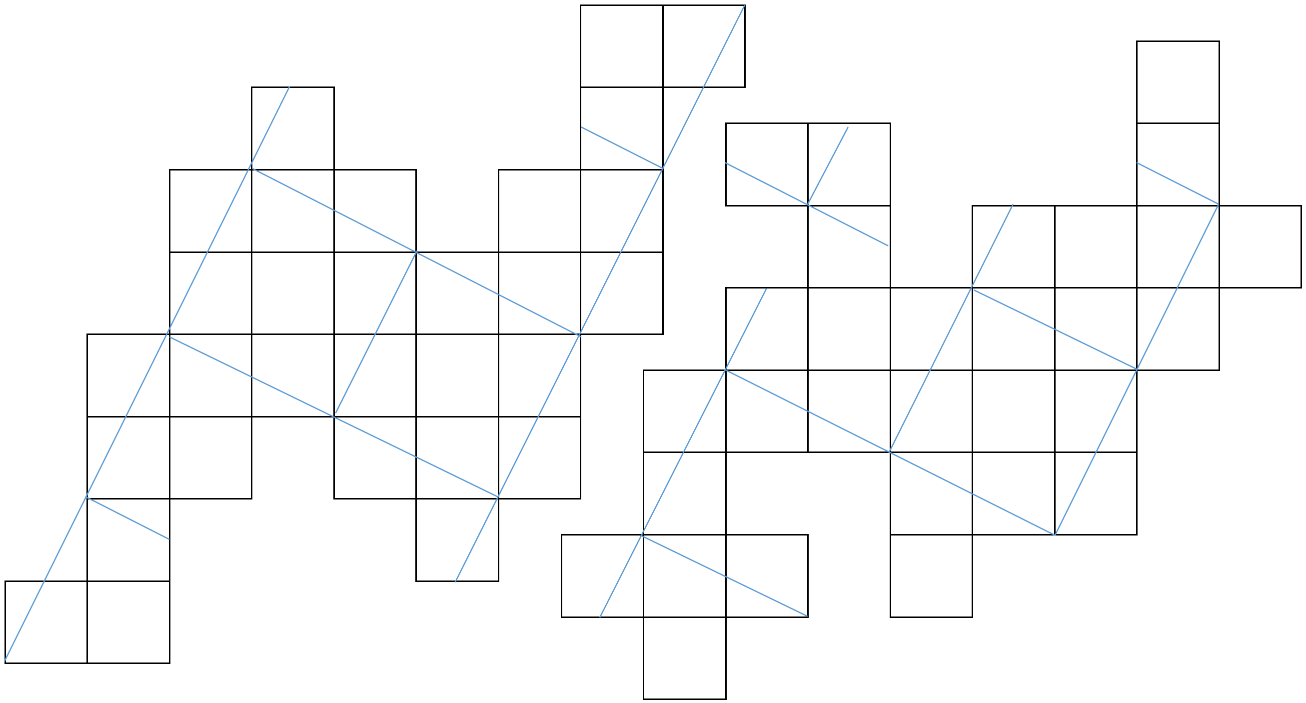


(Note: There are 2263 common developments of area **22** of two boxes of size $1\times 1\times 5$ and $1\times 2\times 3$.)

Are there common developments of 4 or more boxes?
(Is there any upper bound of this number?)

October 23, 2012: Email from Shirakawa…

"I found polygons of area 30 that fold to 2 boxes of size 1 × 1 × 7 and √5 × √5 × √5. This area allows to fold of size 1 × 3 × 3, it may be the smallest area of three boxes if you allow to fold along diagonal."



Surface areas and possible size of boxes

If you want to find common developments of three boxes,

If you want to find common developments of four boxes,

Area	3-tuples	Ar[illegible]	3-tuples
22	(1,1,5),(1,2,3)	46	(1[illegible],11),(1,2,7),(1,3,5)
30	(1,1,7),(1,3,3)	70	(1,1,17),(1,2,11),(1,3,8),(1,5,5)
34	(1,1,8),(1,2,5)	94	(1,1,23),(1,2,15),(1,3,11), (1,5,7),(3,4,5)
38	(1,1,9),(1,3,4)	11[illegible]	[illegible] (1,4,[illegible]),(2,5,7)

Known results

Area 30 was on the edge…

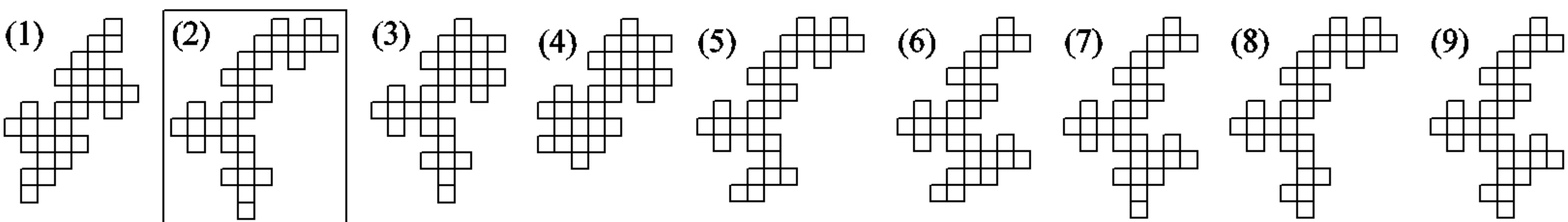
In 2011, Matsui's program based on exponential time algorithm

- enumerated all developments of area 22
  - there are 2263 development of boxes of size 1×1×5 and 1×2×3
- ran in 10 hours on his desktop PC

My student, Dawei, succeeded! on June, 2014, for his master thesis on September ;-)

- We completed enumeration of developments of area 30! [Xu, Horiyama, Shirakawa, Uehara 2015]
- Summary:
  - It took 2 months by Supercomputer (Cray XC 30) in JAIST.
  - There are 1080 common developments of 2 boxes of size 1 × 1 × 7 and 1 × 3 × 3
  - Among 1080, the following 9 can fold to a cube of size √5 × √5 × √5.

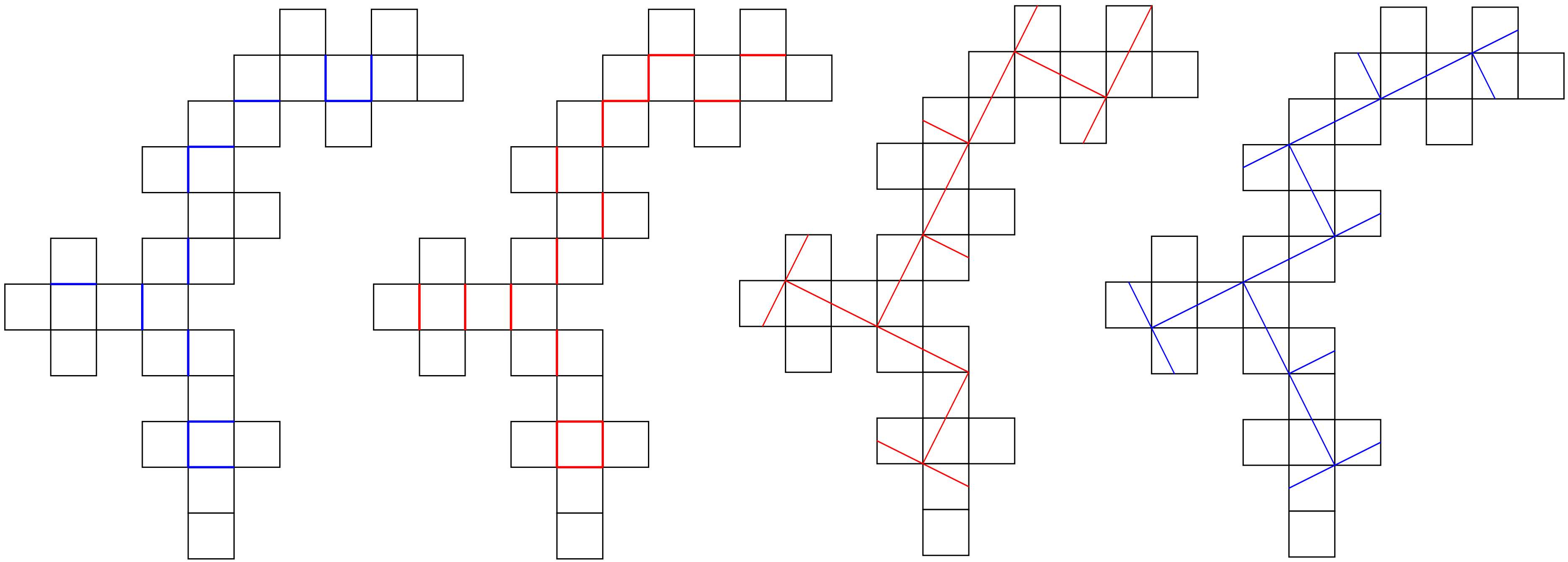
Note: Using BDD, the running time is reduced to 10 days!

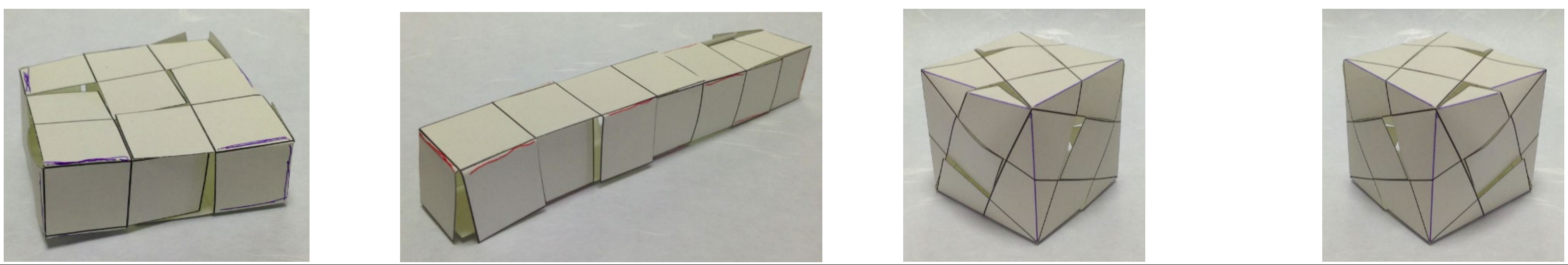


Quite surprisingly, (2) has two different ways for folding the cube!!

Miracle Development

This pattern has 4 ways of folding to box!!

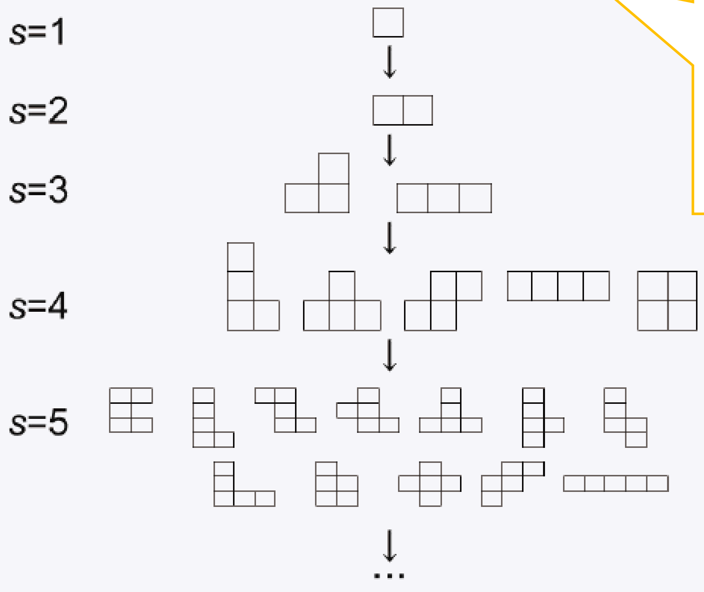




Brief Algorithm for finding them

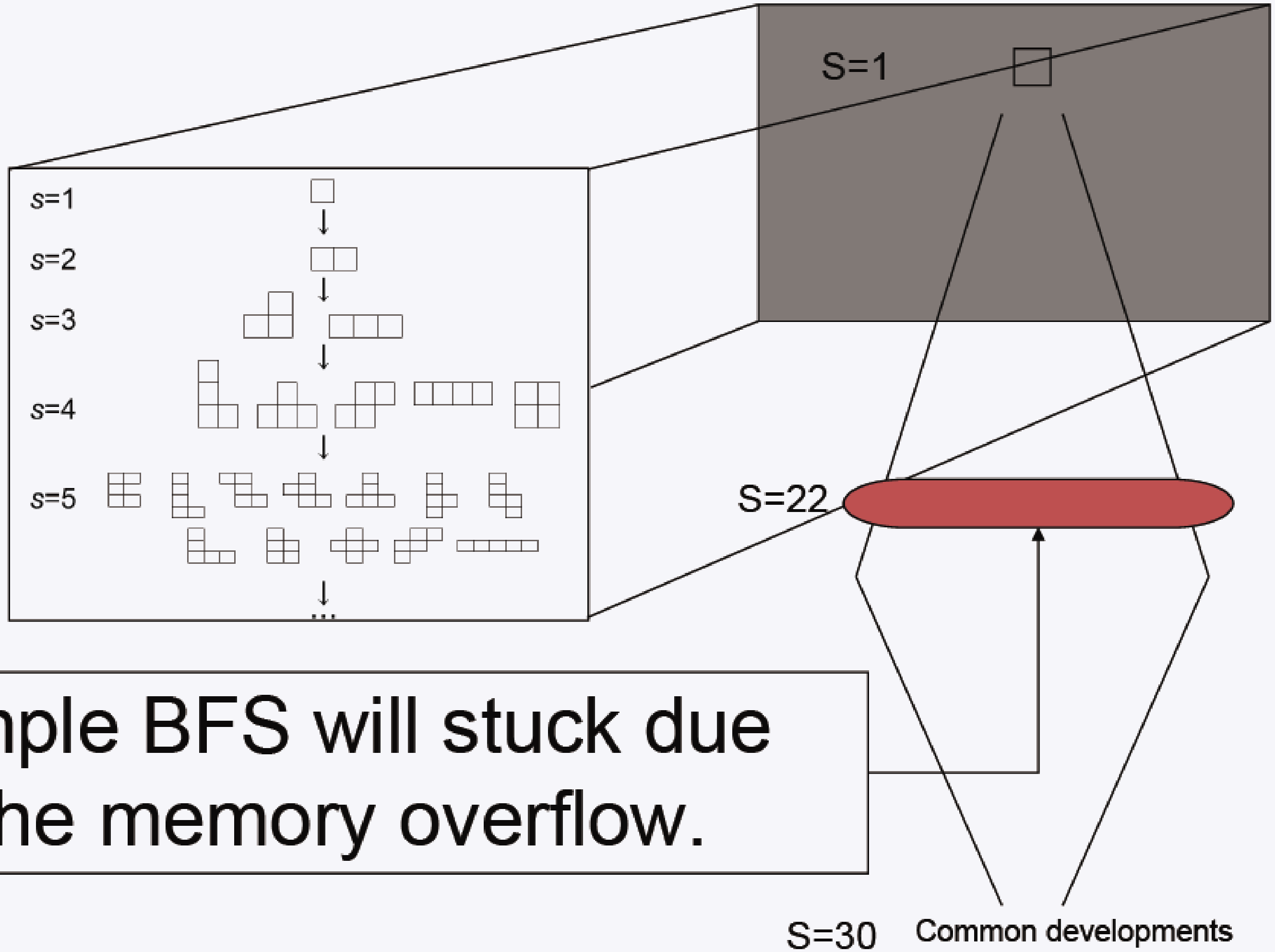
The enumerate approach

- The basic idea is similar to finding two boxes of size 1×1×5 and 1×2×3 [6].
- We start from a single 1 square, then add another square adjacent to it, and extend the set of partial developments, repeat this step, untill 30 squares.



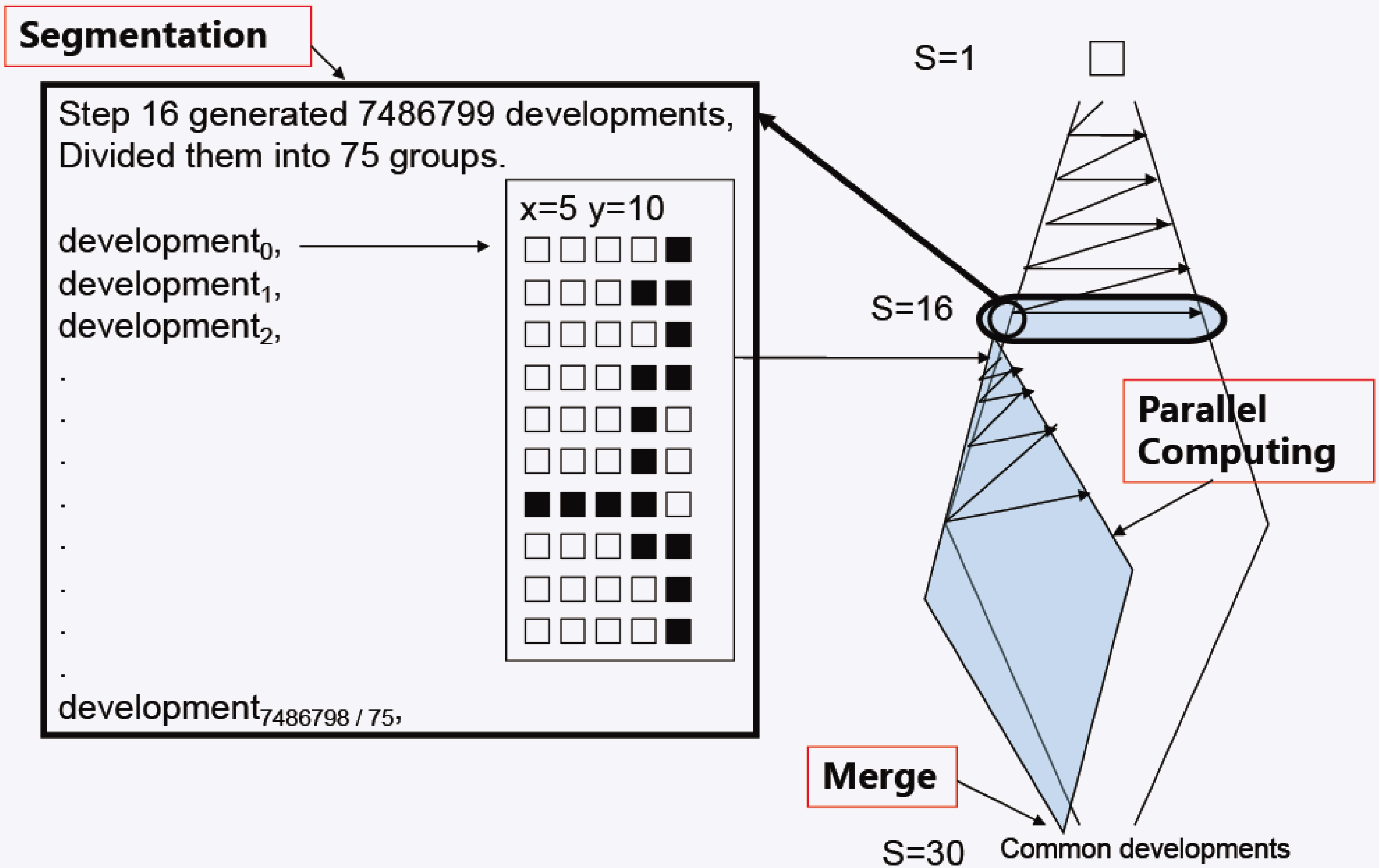
From Ph.D defense slides by Dawei on June 15, 2017

The simple BFS gets stuck



Simple BFS will stuck due to the memory overflow.

Our solution



Summary and future work…

If you want to find common developments of three boxes,

If you want to find common developments of four boxes,

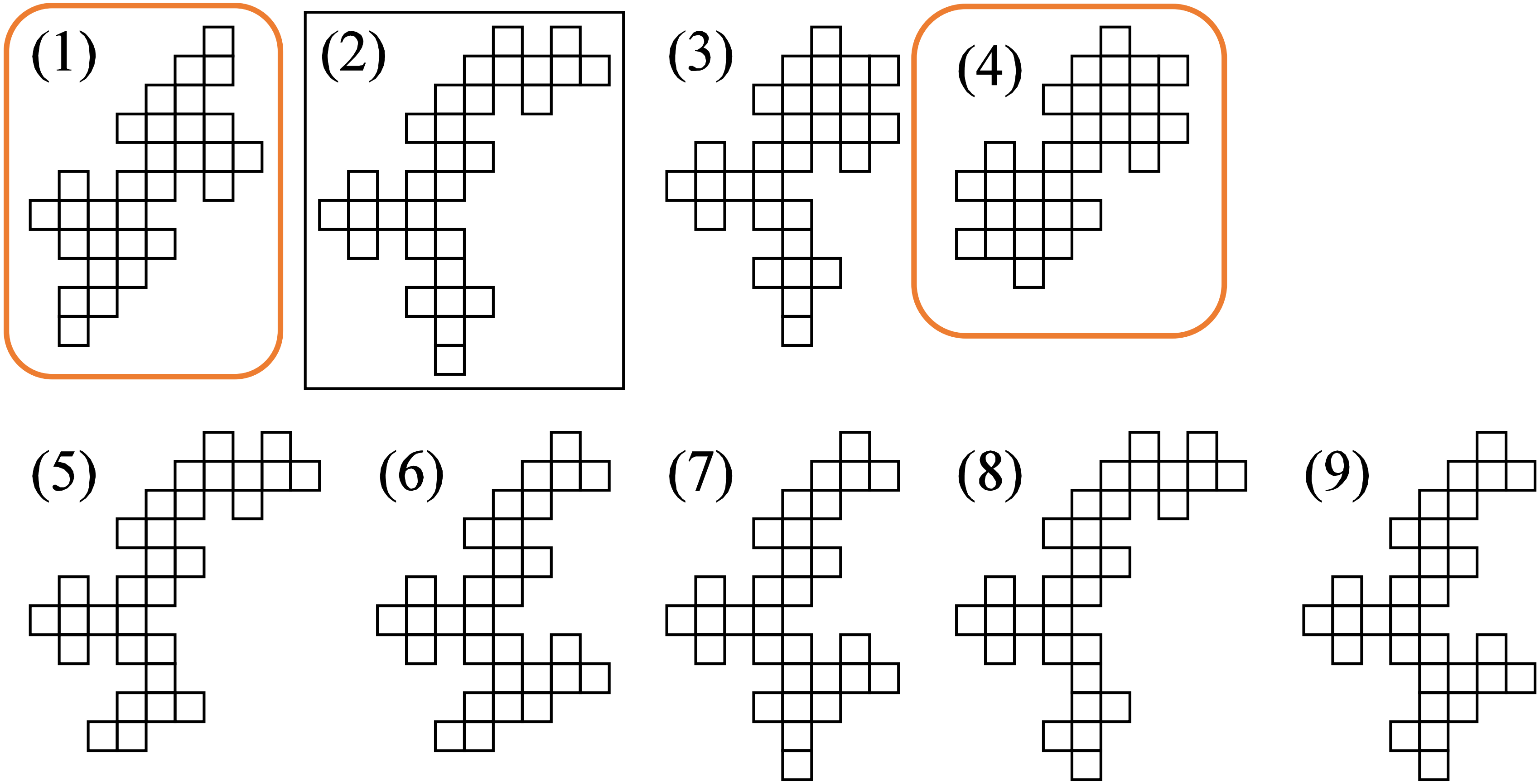
Known results

Area	3-tuples	Area	3-tuples
22	(1,1,5),(1,2,3)	46	(1,1,11),(1,2,7),(1,3,5)
30	(1,1,7),(1,3,3)	70	(1,1,17),(1,2,11),(1,3,8),(1,5,5)
34	(1,1,8),(1,2,5)	94	(1,1,23),(1,2,15),(1,3,11),(1,5,7),(3,4,5)
38	(1,1,9),(1,3,4)	118	(1,1,29),(1,2,19),(1,3,14),(1,4,11),(1,5,9),(2,5,7)

- In 2011, area 22 was enumerated in 10 hours on a desktop PC.
- In 2017, area 30 was enumerated in 2 months by a supercomputer, and improved to 10 days on a desktop PC.
- It seems to be quite hard to area 46 in this approach…

Some progress…?

- We can try more on the symmetric ones…



Some progress…?

- We can try more on the symmetric ones…
  1. The search space can be drastically reduced,
  2. Memory size is reduced into half, and
  3. Area can be incremented by 2.

(Quite sad) NEWS:

No common development of 3 boxes of areas 46 and 54

- Area 46: There are symmetric common developments of two different boxes of any pair of size $1 \times 1 \times 11$, $1 \times 2 \times 7$, and $1 \times 3 \times 5$, but there are no symmetric common development of 3 of them.
- Same as for the area 54 of size $1 \times 1 \times 13$, $1 \times 3 \times 6$, and $3 \times 3 \times 3$.

Open problems

- Are there common developments of 3 boxes of size 46 or 54?
- Is there any common development of 4 boxes?
- Is there any upper bound of k of the number of boxes that share a common development? It is quite unlikely that there is a common development of 10,000 different boxes,,, but who knows?

FYI: The number of different polyominoes is known up to area 45. (by Shirakawa on OEIS)

More open problems

The other variants of the following general problem:

For any polygon P, determine if you can fold to a box Q (or other convex polyhedron)

Known (related) results :

- General polygon P and convex polyhedron Q, there *is* a pseudo poly-time algorithm, however, …
  - It runs in $O(n^{456.5})$ time! (Kane, et al, 2009)
- When Q is a box of size a × b × c, n-gon P, and edge-gluing is given,
  - O((n+m)log n) time algorithm
  - Parameter m indicates "how many line segments contained in an edge of Q" [Horiyama, Mizunashi 2017]
  - Open: a,b,c are not given.

There are many open problems, and young researchers had been solving them ☺