Introduction to Algorithms and Data Structures

Lesson 3: Searching (1) Sequential search

Professor Ryuhei Uehara,
School of Information Science, JAIST, Japan.
uehara@jaist.ac.jp
http://www.jaist.ac.jp/~uehara

How to tackle the problem

- Consider data structure and how to store data
  - Data are in an array in any ordering
  - Data are in an array in increasing order
- Search algorithm: The way of searching
  - Sequential search
  - m-block method
  - Double m-block method
  - Binary search
- Analysis of efficiency
  - Big-O notation

Search Problem

- Problem: S is a given set of data. For any given data x, determine efficiently if S contains x or not.

- Efficiency: Estimate the time complexity by $n = |S|$, the size of the set S
  - In this problem, "checking every data in S" is enough, and this gives us an upper bound O(n) in the worst case.

Roughly, "the running time is proportional to n."

Data structure 1
Data are stored in arbitrary ordering

- Each element in the set *S* is stored in an array s from s[0] to s[*n*-1] in any arbitrary ordering.

s[ ]=	37	12	25	9	87	33	65	3	29

Sequential search

- Input: any natural number *x*
- Output:
  - If there is i such that s[i] == *x*, output i
  - Otherwise, output -1 (for simplicity)

```
for (i=0; i<n; ++i)
   if(x==s[i]) return i;
return -1;
```

In the worst case, we need n comparisons.
Thus, the running time is proportional to n.
$\rightarrow$ O(n) time algorithm

Precise time complexity of sequential search

- At most 3n + 2 steps

Initialization of i takes 1 operation

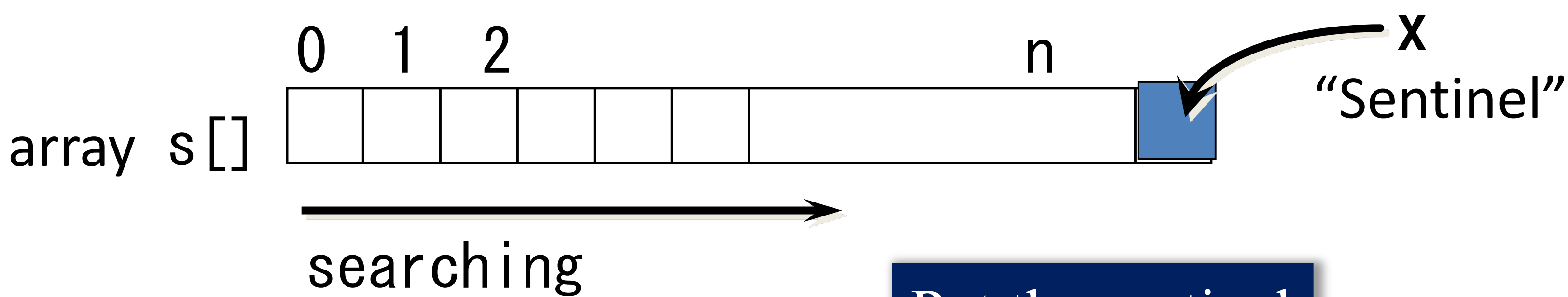
```
for (i=0; i<n; ++i)
   if(x==s[i]) return i;
return -1;
```

For the number of loops ≦ n,
  comparison ×2 (==, <)
  increment ×1 (++)

Return takes 1 operation

Programming tips 1: simplify by using "sentinel"

Before searching, push *x* itself at the end of the array;
Then you definitely have `x==s[i]` for some `0<=i<=n`
So you do not need the check `i<n` any more.




```
s[n] = x;
i = 0;
while(x != s[i])
 i = i+1;
if(i < n) return i;
else      return -1;
```

Put the sentinel

Simple loop!
→ 2 operations

At most 2n+4 operations
$=O(n)$

Analysis of the number of comparisons

```
s[n] = x;
i = 0;
while(x!=s[i])
 i = i+1;
if(i < n)
  return i;
else
  return -1;
```

- The best case: 1 time
  - In the case of s[0] == x
- The worst case: n times
  - x is not in s[0]…s[n-1]
- The average case: $\sum_{i=1}^{n+1} \frac{i}{n} = \frac{n+2}{2}$
  - The expected value of # of comparisons
  - The i-th element is compared with probability 1/n
  - The number of comparisons when x is equal to the i-th element is i.

Randomized algorithm

Flip a fair coin, and
- “H”: search from s[0] forwardly
- “T”: search from s[n−1] backwardly

Intuition:

For any (sometimes fixed or unbalanced) input, the average case occurs on average.

RANDOMIZED ALGORITHM

The behavior depends on random numbers.
The worst case occurs with low probability.

Data structure 2
Data in the array in increasing order

- s[]=

3	9	12	25	29	33	37	65	87

- Q: Any improvement in sequential algorithm?

```
s[n]=x;
i = 0;
while(x!=s[i])
 i = i+1;
if(i < n) return i;
else      return -1;
```

We can stop when s[i] is greater than x
x!=s[i] → x>s[i]

Data structure 2
Data in the array in increasing order

- s[]=

3	9	12	25	29	33	37	65	87

- Q: Any improvement in sequential algorithm?

```
s[n]=x;
i = 0;
while(s[i]<x)
 i = i+1;
if(i < n) return i;
else      return -1;
```

We can stop when s[i] is greater than x
x!=s[i] ➔ x>s[i]

Data structure 2
Data in the array in increasing order

- s[]=

3	9	12	25	29	33	37	65	87

- Q: Any improvement in sequential algorithm?

```
s[n]=x;
i = 0;
while(s[i]<x)
 i = i+1;
if(i < n) return i;
else      return -1;
```

We can stop when s[i] is greater than x
x!=s[i] ➔ x>s[i]

It may stop even if i<n
i<n ➔ s[i]==x

Data structure 2
Data in the array in increasing order

- s[]=

3	9	12	25	29	33	37	65	87

- Q: Any improvement in sequential algorithm?

```
s[n]=x;
i = 0;
while(s[i]<x)
 i = i+1;
if(s[i]==x) return i;
else       return -1;
```

We can stop when s[i] is greater than x

x!=s[i] ➔ x>s[i]

It may stop even if i<n

i<n ➔ s[i]==x

Data structure 2
Data in the array in increasing order

- s[]=

3	9	12	25	29	33	37	65	87

- Q: A ... equential algorithm?

When x is not in s[], it returns n
s[n]=x ➔ s[n]=x+1

```
s[n]=x;
i = 0;
while(s[i]<x)
 i = i+1;
if(s[i]==x) return i;
else        return -1;
```

We can stop when s[i] is greater than x
x!=s[i] ➔ x>s[i]

It may stop even if i<n
i<n ➔ s[i]==x

Data structure 2
Data in the array in increasing order

- s[]=

3	9	12	25	29	33	37	65	87

- Q: A equential algorithm?

When x is not in s[], it returns n
`s[n]=x` → `s[n]=x+1`

```
s[n]=x;
i = 0;
while(s[i]<x)
 i = i+1;
if(s[i]==x) return i;
else        return -1;
```

We can stop when s[i] is greater than x
`x!=s[i]` → `x>s[i]`

It may stop even if `i<n`
`i<n` → `s[i]==x`

Data structure 2

Data in the array in increasing order

- s[]=

3	9	12	25	29	33	37	65	87

- Q: A equential algorithm?

When x is not in s[], it returns n
s[n]=x ➔ s[n]=x+1

```
s[n]=x+1;
i = 0;
while(s[i]<x)
 i = i+1;
if(s[i]==x) return i;
else        return -1;
```

We can stop when s[i] is greater than x
x!=s[i] ➔ x>s[i]

It may stop even if i<n
i<n ➔ s[i]==x

Data structure 2
Data in the array in increasing order

- s[]=

3	9	12	25	29	33	37	65	87

  - Exit from loop when: s[i]≧x
  - Check after loop: s[i]==x
  - Sentinel: greater than x, e.g., x+1

```
s[n]=x+1;
i = 0;
while(s[i]<x)
 i = i+1;
if(s[i]==x) return i;
else        return -1;
```

Q. Improve of comparison?

A. Average is better. But the same in the worst case