Introduction to Algorithms and Data Structures

Lesson 1: Foundation of Algorithms (1) Basic Models

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Summary

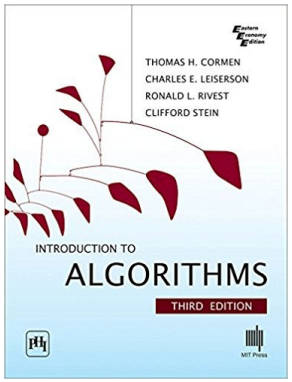
Introduction to Algorithms and Data Structures

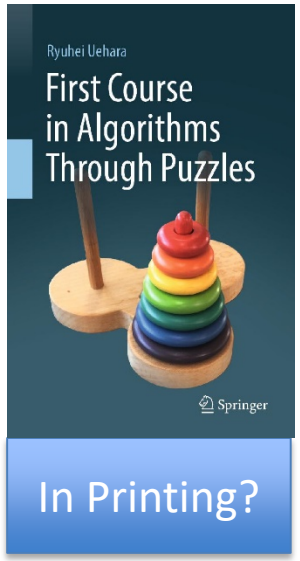
- By Professor Ryuhei Uehara, JAIST
- Goal: Understanding of *value* of Algorithms

- An **algorithm** is a **way/method for solving a problem.**
- A **data structure** is a **way/method for storing data** in a computer.
- In general, for a problem, there are many combinations of algorithms and data structures. We need to evaluate them according to there efficiency, and choose the best one.
- However, the important point is that to master the way of thinking of algorithm design.
- In this short course, we learn several basic and representative problems and algorithms for them. We analyze their correctness and efficiency.

References

- Textbook
  - "Introduction to Algorithms, 3rd ed." Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest, Clifford Stein, 2010, MIT Press.
  - "First Course in Algorithms through Puzzles," Ryuhei Uehara, in printing, Springer, 2019.





Requirements

- No special knowledge is required, but…
  - It is better to have some experience of programming

    …in any programming language
    - C, C++, Java, C#, Ruby, Python, Scheme, Haskell, …

  - **Algorithm** itself is independent from any programming language.
    - I will use so-called “pseudo-code” to describe high-level idea of an algorithm.

What algorithm is…

An abstract description of method for solving a problem using a computer.

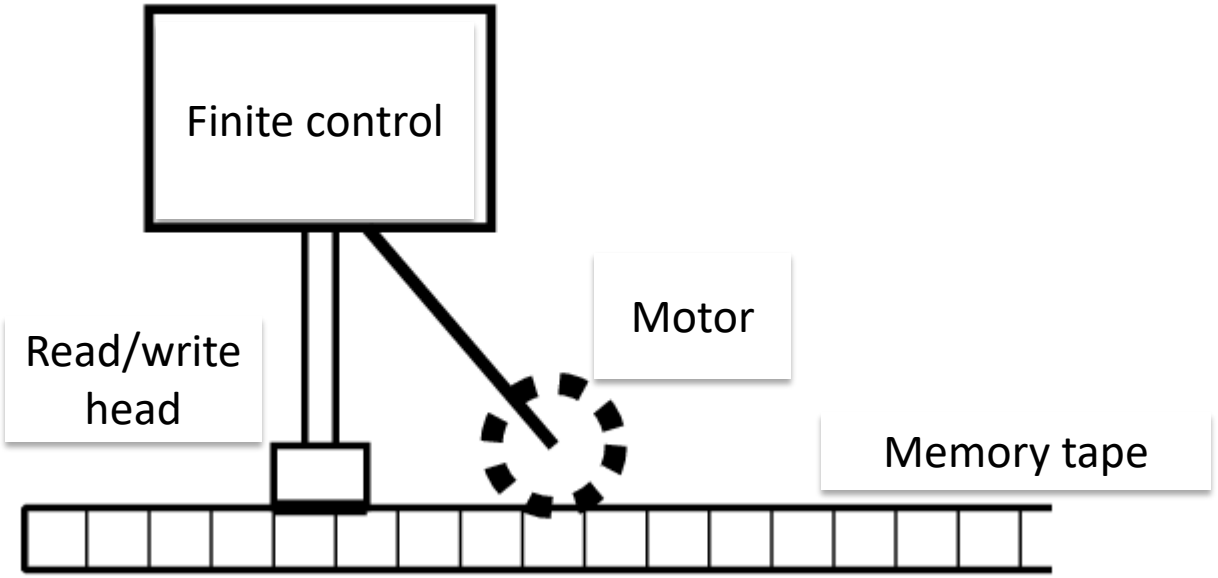
- What "solving a problem" means;
  - We can obtain a correct answer for any input
  - It can be obtained with reasonable costs;
    - Computation is done in a polynomial time of the length of an input
    - in a polynomial space (=memory) of the length of an input
- A problem is "unsolvable" if
  - it takes so long time for some inputs,
  - it takes so much memories for some inputs, or
  - (we cannot make any program for the problem)

Model of Computing

How does "computer" work?
What is a "computation"?

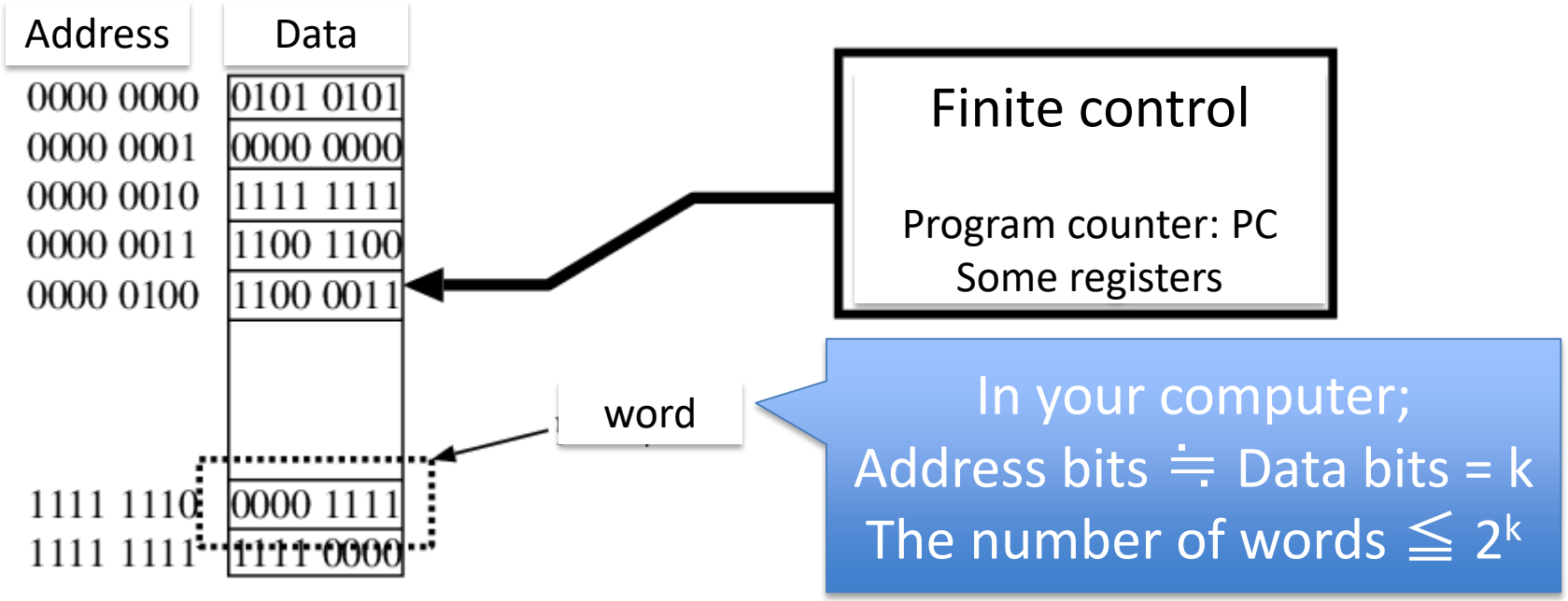
- Description/efficiency of an algorithm are different depending on a model of computation.
  - What "basic operations" are?
  - What kind of data in memory?
    - Natural numbers, real numbers (with infinite accuracy?), images, music data…?
- There are some standard models of computing
  - Turing machine: The mathematical model by Alan Turing. Base of all arguments of computation.
  - RAM model: a standard model when we consider algorithms.

Turing machine model



- Quite simple mathematical/theoretical model.
- Turing prove that a Turing machine is “universal”, which means that every computable function can be computed by a Turing machine.
- Turing machine is toooooo simple to do programming in a real world
  - Few basic operations like +, -, *, /, and so on…
  - It is not good for discussion of “algorithms”

RAM Model
(Random Access Memory)



- It consists Memory and CPU (Central Processing Unit)
  - We do not mind Input/Output
- It is essentially the same as your computer
- CPU can access any address randomly (not sequentially) in a unit cycle
- Programming language C is a system that show you this structure implicitly (like arrays and pointers)

Programming Language

- Compiler translates any “readable” program (for human) to an executable file in machine language (for the CPU)
- E.g. Programming language C; It is okay if you know…
  1. variable
  2. array
  3. pointer
  4. control statement (e.g., if, while)
  5. recursive call

Basic of C: Hello World

- Display “Hello World” on screen

```
#include <stdio.h> /* for printf*/

main(){
    printf(“Hello World”);
}
```

statement

Semi-colon at the end of a statement

“Algorithms” do not depend on programming language, but we need some agreement in this class.

Basic of C: Mathematics

- Mathematical operations: `+`, `-`, `*`, `/`

Equation	meaning
3+4	Add 3 and 4
3-1	Subtract 1 from 3
3*3	Multiply 3 and 3
4/2	Divide 4 by 2

  - We do not mind if they are integers (`int`, etc.) or real numbers (`float`, `double`, etc.) in this class

Note: For C beginner

- integer/integer is an integer
  - Ex: `1/3 is 0, and 1.0/3 is 0.3333…`
- You can use () for control of the order of operations
  - You cannot use {} and [] in mathematical formula
  - Ex: `{(3+4)*3+4}*6` is not valid. You have to write `((3+4)*3+4)*6`
- No power operations (you have some library of functions to compute it)

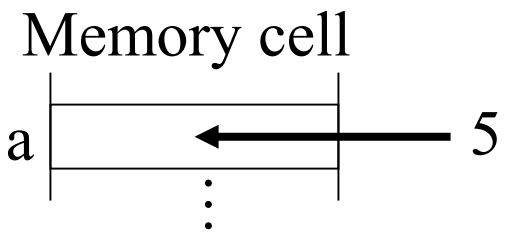
Basic of C: Variable

- Variable: It is a memory cell, that indicates the “place” to memory a result of computation
- Rules for naming
  - Start with alphabet (UPPER, lower letters, and _)
  - From the second letter, you can use alphabets and numbers
    - Not any other
  - Upper and lower letters are different
    - FF, ff, fF, and Ff are all different names
  - Not reserved words in C (e.g., main, include, return)

  - Good: `x`, `orz`, `T_T`, `IE9`, `projectX`, `ff4`, `y2k`, `JAIST`
  - Bad: `7th`, `uehara@jaist`, `ac.jp`, `tel#`

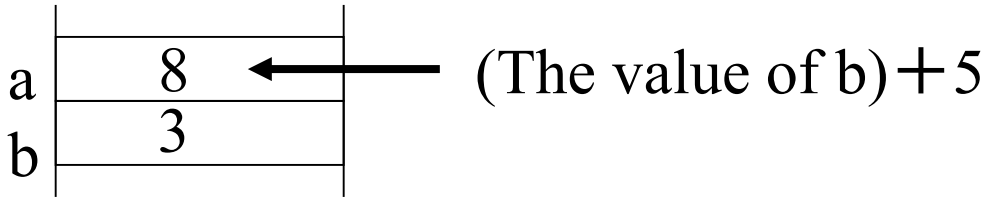
Basic of C: Assignment statement

- a=5

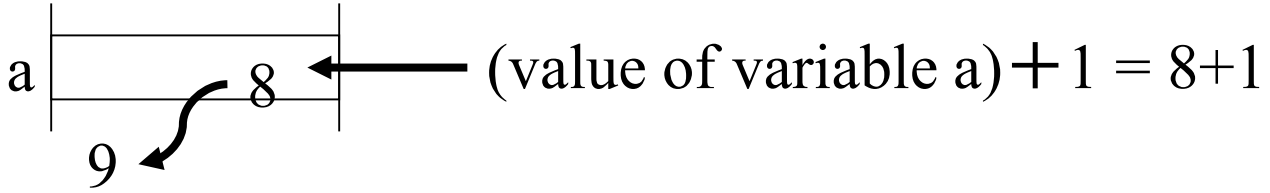


"=" is not "equal" in the sense of mathematics

  - Store the value 5 to the place named by a in memory
- a=b+5



  - Store value of "value stored at the place named by b (or value of the variable b) plus 5" to the place named by a
- a=a+1



  - Store value of "the value of variable a plus 1" to the place named by a

Basic of C: declaration of variable

- You have to declare variables beforehand (in C language)

Good

```
main(){
   int a,b;
   a = 5; b = 3;
   printf("a+b=%d",a+b);
}
```

Variables a and b in integer

Bad

```
main(){
   a = 5;
   printf("%d",a);
}
```

It is not good!

Note: Recent language (like python) does not require to declare beforehand.

Basic of C: Mathematical functions

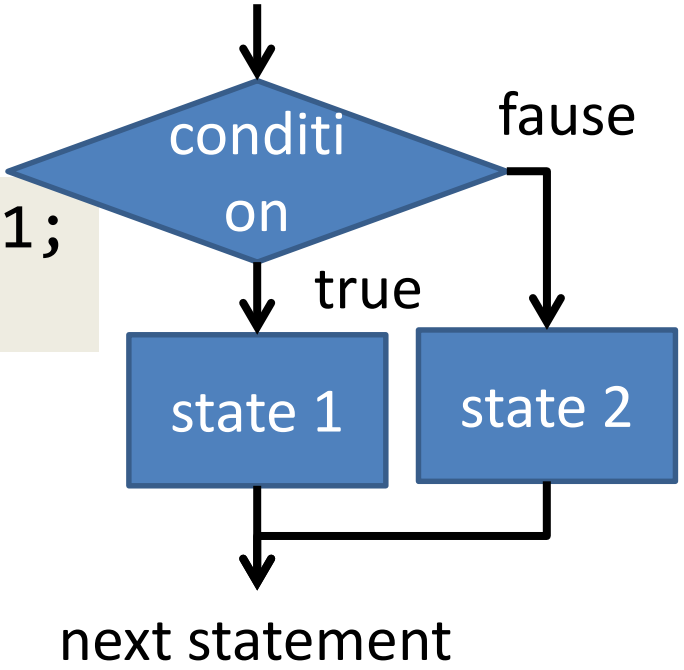
	function	Math. symbol	type	Parameter type
Square root	sqrt(x)	$\sqrt{x}$	double	double
Power	pow(x, y)	x^y	double	double
Logarithm	log(x)	$\log_e x$	double	double
Logarithm	log10(x)	$\log_{10} x$	double	double
Exponential	exp(x)	e^x	double	double

- Source code: include the following header file `#include <math.h>`
- Compile: Option -lm is required
  - gcc main.c –lm

Basic of C: Control statements
if statement – conditional branch

- Grammar

```
if (condition) state 1;
else state 2;
```



If condition is true, perform statement 1, and perform statement 2 if it is false

– Ex: Output EVEN if n is even, and ODD if it is odd.

```
if(n%2==0) printf("EVEN");
else printf("ODD");
```

Basic of C: Representations of conditions (1/2)

symbol	meaning	example	meaning of example
==	equal	n == 2	n is equal to 2
!=	not equal	n != 0	n is not equal to 0
>	greater than	n > 3	n is greater than 3
>=	g.t. or equal	n >= 3	n is g.t. or equal to 3
<	less than	n < 0.01	n is less than 0.01
<=	l.t. or equal	n <= 0.01	n is l.t. or equal to 0.01
&&	and	0 < n && n <= 10	n is greater than 0 and less than or equal to 10
||	or	n < 0 || 0 < n	n is less than 0 or greater than 0
!	not	!(n < 0.01)	n is not less than 0.01

Basic of C: Representations of conditions (2/2)

- You cannot compare 3 or more items
  - `0<x<5` ➔ `0 < x && x < 5`
  - `a==b==c` ➔ `a == b && b == c`

- Example: Check of the leap year
  - Dividable by 400, or
  - Not dividable by 100 but dividable by 4

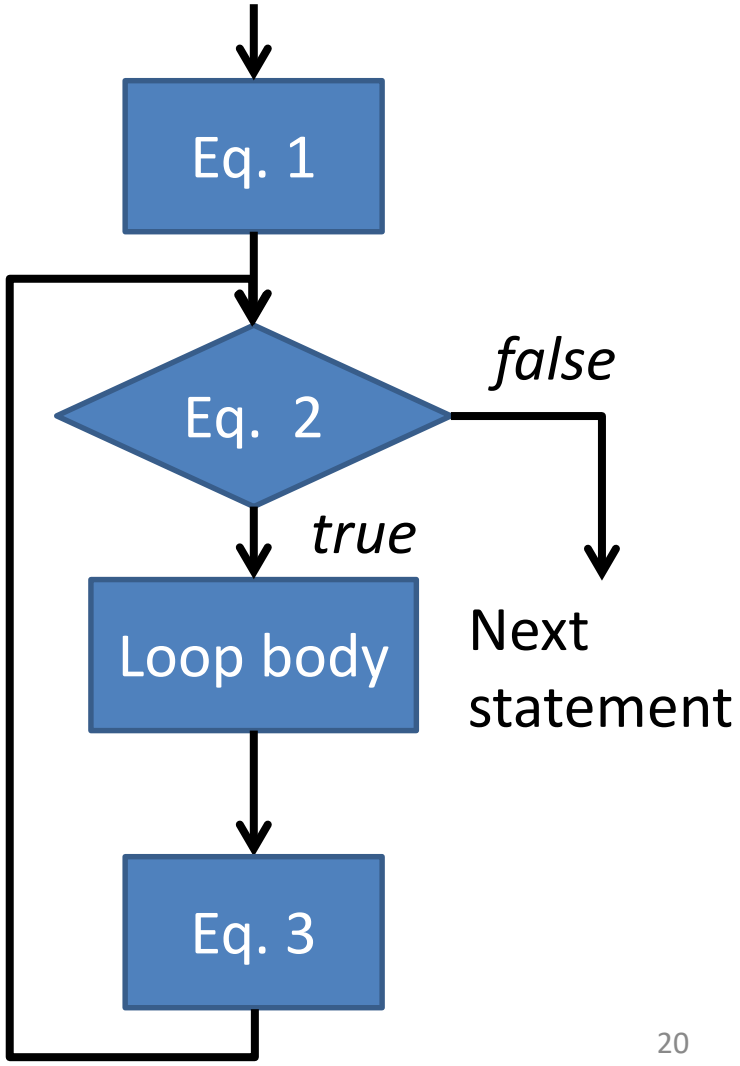
```
year%400==0 || (year%100!=0 && year%4==0)
```

Basic of C: Control statements for loop – repeating (1/4)

- Grammar

```
for(eq.1;eq.2;eq.3){
  loop body
}
```

- It runs as follows:
  A) Execute eq. 1
  B) If eq.2 is *true*, step C, and step D if *false*
  C) Perform loop body and eq. 3, jump to B
  D) Go to next statement



Basic of C: Control statements for loop – repeating (2/4)

Example: Output the sum $\sum_{i=1}^{n} i$ between 1 to n

```
int i,n,sum;
n=/*initialized somehow*/;
sum=0;
for(i=1;i<=n;i=i+1){
  sum=sum+i;
}
printf("1+…+%d=%d",n,sum);
```

Basic of C: Control statements for loop – repeating (3/4)

Example: Output the sum $\sum_{i=1}^{n} i^2$ between 1 to n

```
int i,n,sum;
n=/*initialized somehow*/;
sum=0;
for(i=1;i<=n;i=i+1){
  sum=sum+i*i;
}
```

Basic of C: Control statements for loop – repeating (4/4)

- Ex: Compute $\sum_{i=1}^{n}(2i-1)^2$

```
int i,n,sum;
n=/*initialized somehow*/;
sum=0;
for(i=1;i<=2n-1;i=i+2){
  sum=sum+i*i;
}
```

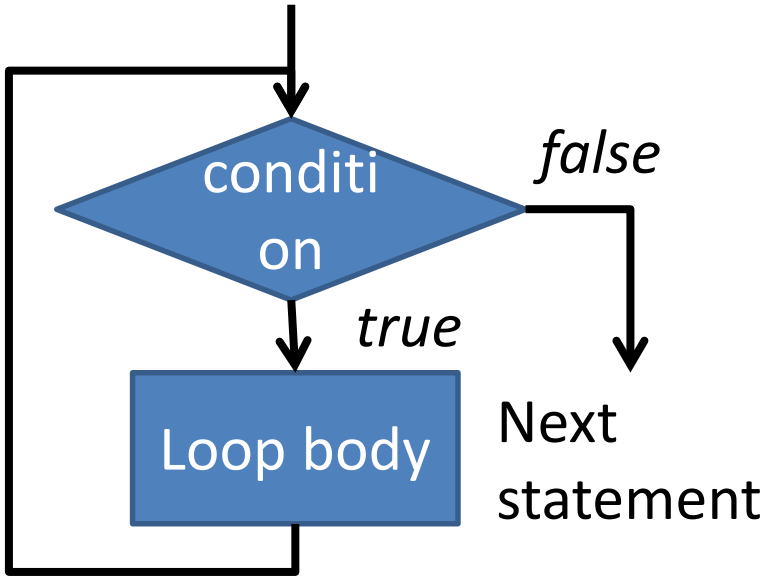
i indicates 2j-1

- Why is this correct?
  - Because; $\sum_{i=1}^{n}(2i-1)^2 = 1^2 + 3^2 + \cdots + (2n-1)^2$

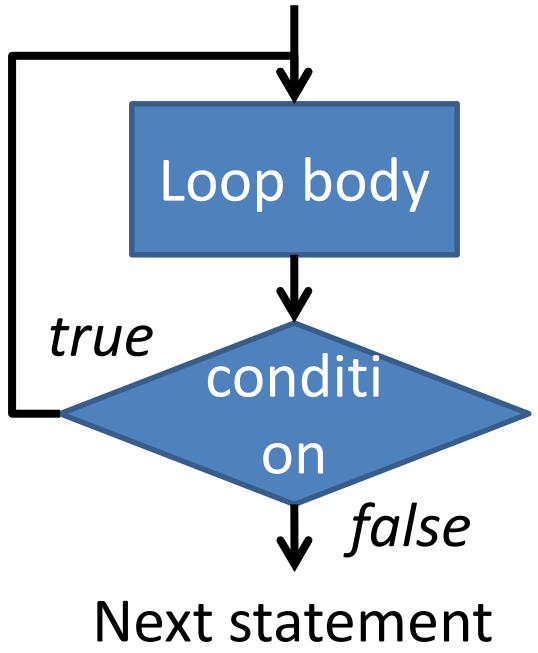
Basic of C: Control statements while loop & do-while loop (1/2)

- Grammar

```
while(condition){
  loop body
}
```




```
do{
  loop body
}while(condition)
```



Basic of C: Control statements while loop & do-while loop (2/2)

Ex: Compute GCD(a,b) of two integers a and b

```
int a,b,r;
a=/*some value*/;
b=/*some value*/;
do{
  r = a % b;
  a = b; b = r;
}while(r!=0);
printf("G.C.D.=%d",a);
```

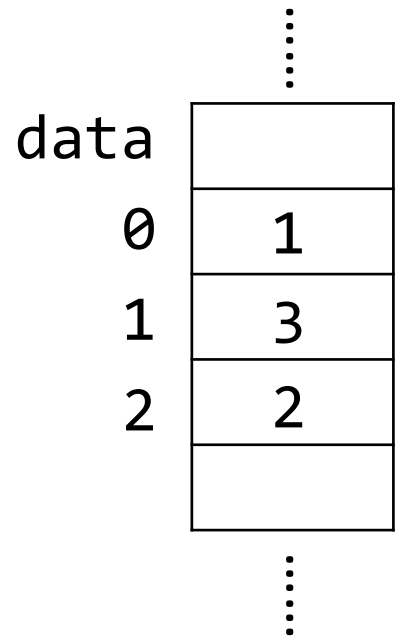
Ex: a=1848, b=630

a	b	r=a%b
1848	630	588
630	588	42
588	42	0
42	0	0

This method (algorithm) is known as "Euclidean mutual division method"

Basic of C: Array (1/2)

- What is array?
  Data structure that aligns many data in the same type (`int`, `float`, etc.) sequential in memory
- Ex: `int data[3]`
  - `3` consecutive memory cells are kept as name “data”, in which each cell stores an integer.




```
int data[3];
data[0]=1;
data[2]=2;
data[1]=3;
```

Basic of C: Array (2/2)
Get the maximum

- Ex: compute the maximum value in integer data[100]


```
int data[100];
int i,max;
/*data is initialized somehow*/
max=0;
for(i=0;i<100;i=i+1){
  if(max<data[i]) max=data[i];
}
printf("maximum data = %d",max);
```

Q: Is this program correct?

Basic of C: Array (2/2)
Get the maximum

- Ex: compute the maximum value in integer data[100]


```
int data[100];
int i,max;
/*data is initialized somehow*/
max=0;
for(i=0;i<100;i=i+1){
  if(max<data[i]) max=data
}
printf("maximum data = %d",max);
```

When all `data` is negative, it outputs 0 as the maximum!

Q: Is this program correct?

Basic of C: Array (2/2)
Get the maximum

- Ex: compute the maximum value in integer data[100] – make it correct

```
int data[100];
int i,max;
/*data is initialized somehow*/
max=data[0];
for(i=1;i<100;i=i+1){
  if(max<data[i]) max=data[i];
}
printf("maximum data = %d",max);
```

The value of max is always in data

Report Problem 1.

- Definition of ExOR $\oplus$:
  - $0 \oplus 0=0,\ 0 \oplus 1=1,\ 1 \oplus 0=1,\ 1 \oplus 1=0$
- For integers in binary system, we apply ExOR bitwise; for example,
  - $10_{10} \oplus 7_{10} = 1010_2 \oplus 111_2 = 1101_2 = 13_{10}$

"Exclusive OR" operation

1. Compute the following
   1. $8_{10} \oplus 3_{10}$
   2. $15_{10} \oplus 7_{10}$

Report Problem 1.

2. What does this function S(x,y) do?

```
S(int x, y) {
  x=x ⊕ y;
  y=x ⊕ y;
  x=x ⊕ y;
}
```

Hint: Try to compute
(x=8, y=3),
(x=15, y=7),
(x=1, y=128),
and so on…

- Write your student ID, name, and answer, (and any comment is welcome ☺) in one sheet of paper of A4 size, and submit it tomorrow, 13:00.