I111E Algorithms and Data Structures

2019, Term 2-1

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Assigned: October 28 (Mon)

Deadline: November 11 (Mon), 10:50am

Note: Do not forget to write your name, student ID, problems, and answers on your report. You can send your report by email (to both of us) in PDF file format before the deadline. Reports in Word file formats are not accepted.

Answer some of the following problems in English or in Japanese. Choose problems to make 30 points in total. (If you choose more, we will estimate your score appropriately.)

Problem 1 (5+5 points): We define the bit-wise operation $\oplus$ as follows: $0 \oplus 0 = 0$, $0 \oplus 1 = 1$, $1 \oplus 0 = 1$, and $1 \oplus 1 = 0$. If x and y are two non-negative integers, the operation $x \oplus y$ is defined by applying $\oplus$ to the corresponding digits in the binary representations of x and y.

For example, the result of $5 \oplus 14$ is 11, and it can be computed as follows:

$$\begin{array}{cccccc} & 0 & 1 & 0 & 1 \\ \oplus & 1 & 1 & 1 & 0 \\ \hline = & 1 & 0 & 1 & 1 \end{array}$$

Let x, y be two integer variables whose values are already set. We perform the following substitutions:

$x = x \oplus y;$

$y = x \oplus y;$

$x = x \oplus y;$

(a) Give some concrete values to x and y and check the sequence of substitutions.
(b) What do these substitutions aim at? (Justify your answer with a mathematical proof.)

Problem 2 (5+5 points)

(a) Let $(a_n)_{n \geq 0}$ be the following sequence:

$$a_n = \begin{cases} n+1 & \text{if } n \in \{0,1,2,3\} \\ a_{n-1} + a_{n-4} & \text{otherwise} \end{cases}$$

Describe an efficient algorithm that takes n as input and outputs a_n.
What are the running time and space complexity of your algorithm?

(b) Let $(b_n)_{n \geq 0}$ be the following sequence:

$$b_n = \begin{cases} 0 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ b_{n-1} - b_{n-2} & \text{otherwise} \end{cases}$$

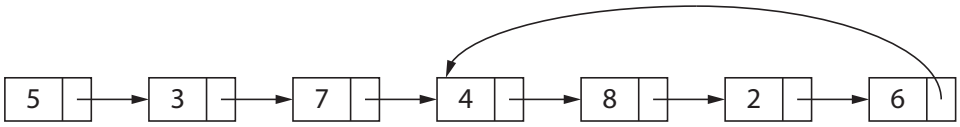
Describe an optimal algorithm that takes n as input and outputs b_n.
What are the running time and space complexity of your algorithm?

Problem 3 (10 points) Let $s[\,]$ be an array of size n with the following property: there exists an (unknown) index m such that the values $s[0]$, $s[1]$, $s[2]$, ..., $s[m]$ are strictly increasing, and the values $s[m]$, $s[m+1]$, $s[m+2]$, ..., $s[n-1]$ are strictly decreasing.

For example, if $s[\,] = \{3, 8, 10, 11, 14, 12, 10, 9, 4, 2, 1\}$, then $m = 4$.

Describe an efficient algorithm that takes $s[\,]$ (and n) as input and outputs m.
What is the running time of your algorithm?

Problem 4 (15 points) You are given a Linked List whose last node's pointer may be NULL or may point to some other node, thus forming a "loop", as shown in the figure:



Note that the sequential search algorithm performed on this Linked List may never terminate.

Describe a linear-time and constant-space algorithm that, given a Linked List as input, determines if it contains a loop or not (by printing "yes" or "no"). Note that the size of the Linked List is unknown. You may assume that the values stored in the nodes are all distinct.

Problem 5 (10 points) Prove that the delete operation on a Binary Search Tree always preserves the BST property (i.e., for every node v, the left subtree contains only nodes that have values lower than v's, and the right subtree contains only nodes that have values higher than v's).

Note

- If you talk about and/or write your report with other students and/or some references, specify them. You may have bad marks if it is not declared.
- If you submit by email, write your name, student id, and subject in the email, and the name of the PDF file should be [s(student ID).pdf]. Submit from the address with domain "jaist.ac.jp".
- If you submit by paper, use A4-size sheets, and staple them at the top-left corner. You may attach a cover or not.