Introduction to Algorithms and Data Structures

Lesson 9: Data structure (3) Stack, Queue, and Heap

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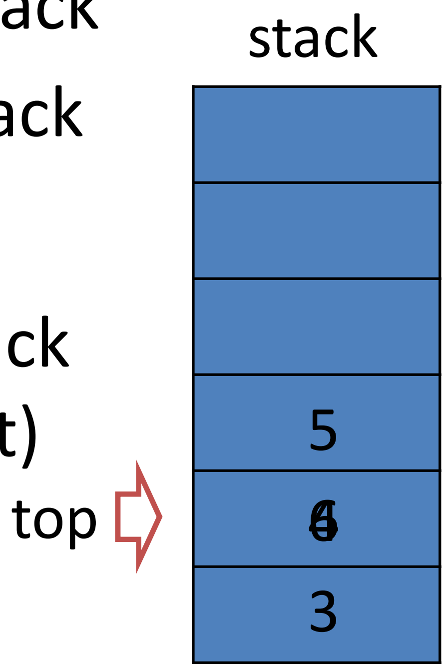
Representative data structure

- Stack: The last added item will be took the first (LIFO: Last in, first out)
- Queue: The first added item will be took the first (FIFO: first in, first out)
- Heap: The smallest item will be took from the stored data

<u>Stack</u>

- The structure that the last data will be popped first (LIFO: Last in, first out)
- Operations
  - push: add new data into stack
  - pop: take the data from stack
- Pointer
  - top: top element in the stack (where the next item is put)



push 3;
push 4;
push 5;
pop; → 5
pop; → 4
push 6;
pop; → 6

Implementation of stack by an array

- Store a data: push(x)

```
stack[top]=x;
top=top+1;
```

- Take the data: pop()

```
top=top-1;
return stack[top];
```

- What kind of errors?
  - Overflow: push (x) when top == size(stack)
  - Underflow: pop(x) when top == 0

Implementation of stack by an array

```
int stack[MAXSIZE];
int top = 0;
void push(int x){
    if(top < MAXSIZE){
        stack[top] = x; top = top + 1;
    } else
        printf("STACK overflow");
}
int pop(){
    if(top > 0){
        top = top - 1; return stack[top];
    } else
        printf("STACK underflow");
}
```

Implementation of stack by linked list

- Point: You don’t need to fix the size of stack

```
typedef struct{
  int data; struct list_t *next;
}list_t;
```

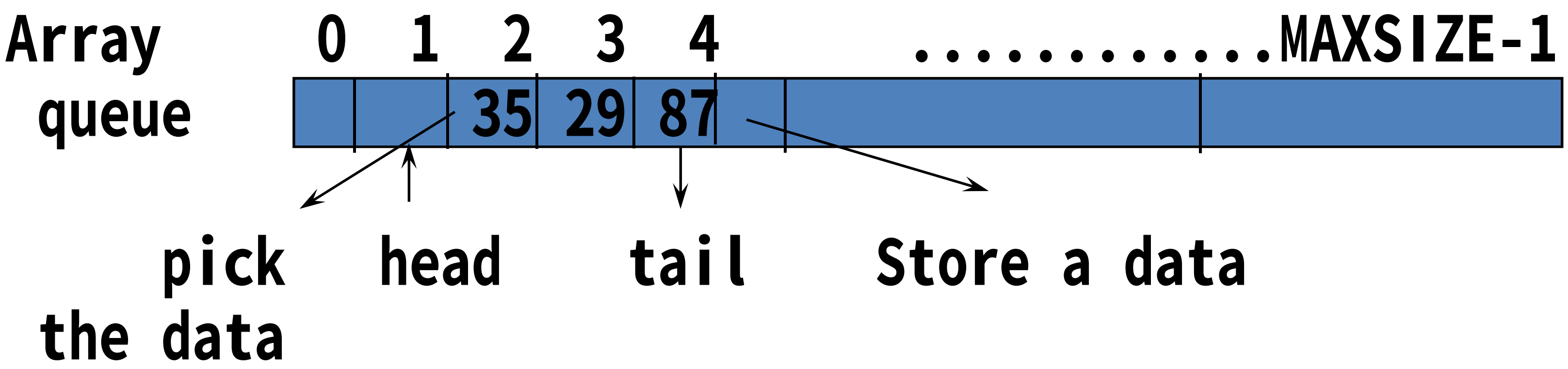
```
list_t* push(list_t *top,int x){
  list_t *ptr;
  ptr=(struct list_t*) malloc(sizeof(list_t));
  ptr->data=x; ptr->next=top; return ptr;
}
list_t* pop(list_t *top){
  list_t *ptr; ptr=top->next; free(top); return ptr;
}
```

It is not necessary if the language has garbage collection

Queue

- The first data will be took first

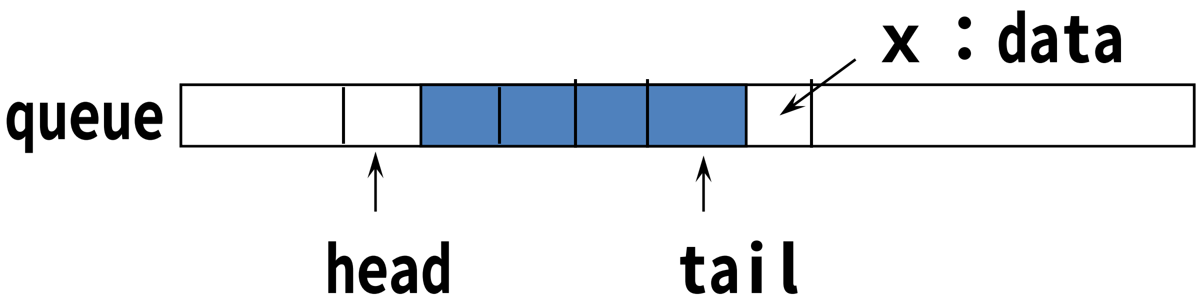
  (FIFO: first in, first out)



Data are stored in

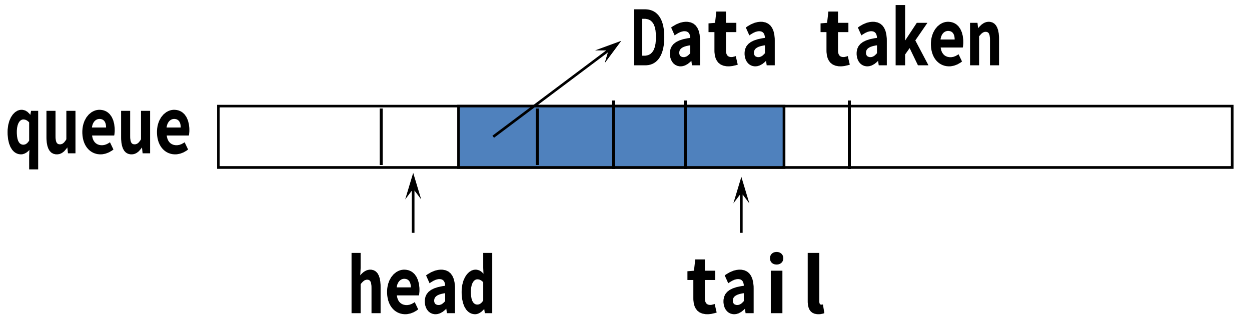
from queue[head+1] to queue[tail]

Add a data into queue




```
void append(int x){
  tail = tail + 1;
  queue[tail] = x;
}
```

Simple implementation of queue by array: take a data




```
int get(){
    head = head + 1;
    return queue[head];
}
```

Problem of simple implementation of queue: Waste area…

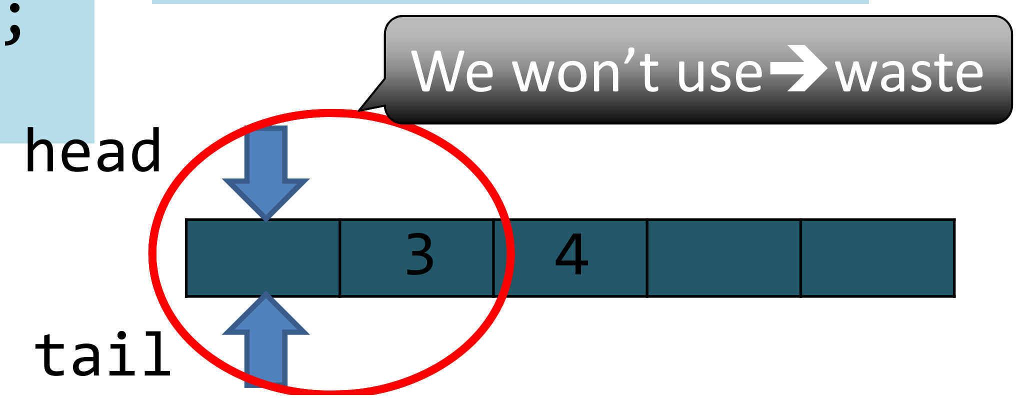
- What happens when we use queue as follows?

```
int get(){
   head = head + 1;
   return queue[head];
}
```

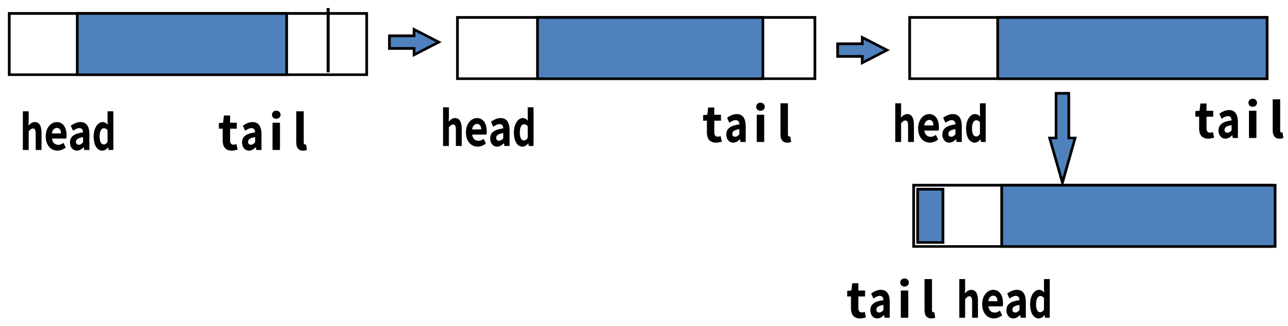
```
int queue[MAX_SIZE];
int head, tail;
void main(){
   head=0; tail=0;
   append(3); get();
   append(4); get();
}
```

```
void append(int x){
   tail = tail + 1;
   queue[tail] = x;
}
```

get() append(4)



Solution: Use array *cyclic*




```
void append(int x){
  tail = (tail + 1) % MAXSIZE;
  queue[tail] = x;
}
int get(){
  head = (head + 1) % MAXSIZE;
  return queue[head];
}
```

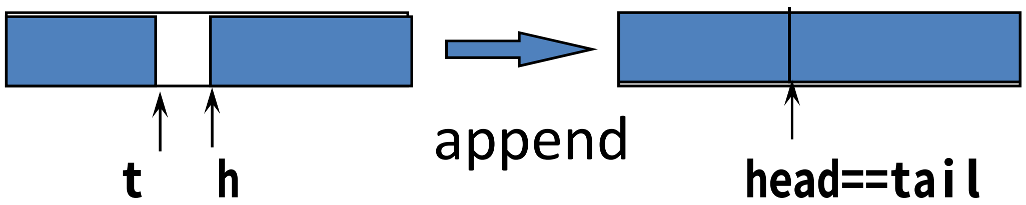
Return to 0

Return to 0

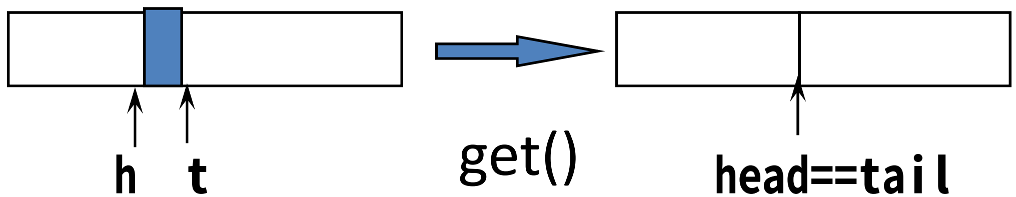
Problem of queue in cyclic array:

We cannot distinguish between (full) and (empty)

When it is full;



When it is empty;



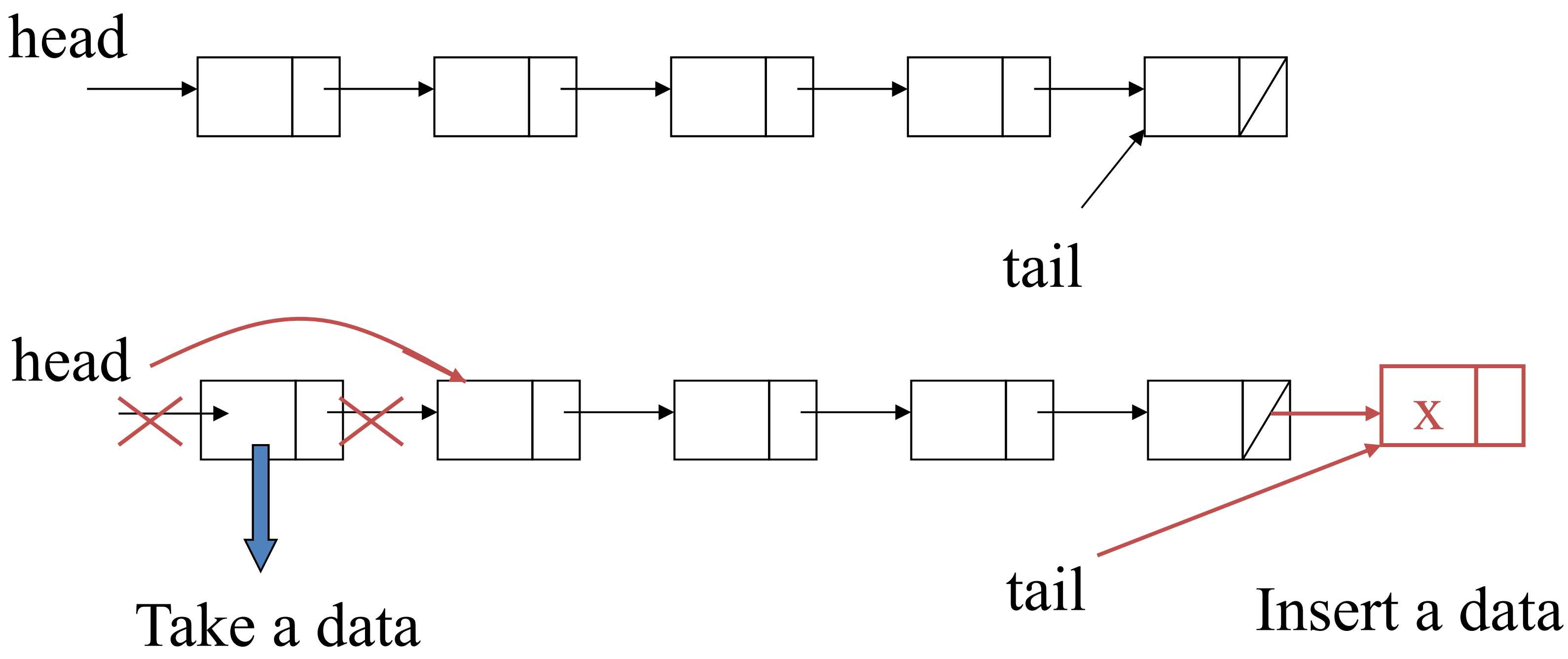
In both cases, we have head==tail.

Solution: We define (full) when we have tail==head when append.

```
void append(int x){
  tail = (tail + 1) % MAXSIZE;
  queue[tail] = x;
  if(tail == head) printf("Queue Overflow ");
}
int get(int x){
  if(tail == head) printf("Queue is empty ");
  else {
    head = (head + 1) % MAXSIZE;
    return queue[head];
  }
}
```

Implementation of queue by linked list

Insertion of a data : From tail of the list: pointer `tail`
Take a data : From top of the list: `pointer head`



Exercise: Make program by yourself!

Heap

- Add/remove data
- Elements can be taken from minimum (or maximum) in order

Q. How can we implement?

Implement of heap (1): Simple implemen

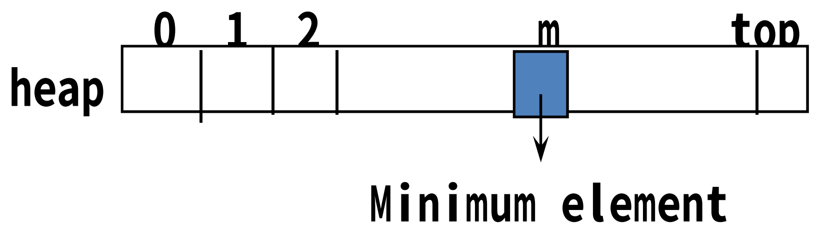
An array `heap[]` and `top`, the number of data

- Initialize: `top = 0`
- Insert data:

```
heap[top] = x;
top = top + 1;
```

- Take minimum one:
  Find the minimum element heap[m] in `heap[]` and output. Then copy `heap[top-1]` to `heap[m]`, and decrease top by 1.

```
m = 0;
for(i=1; i<top; i++)
  if(heap[i] < heap[m])
    m = i;
x = heap[m];
heap[m] = heap[top-1];
top = top - 1;
return x;
```



Problem of simple implementation: Slow for reading data

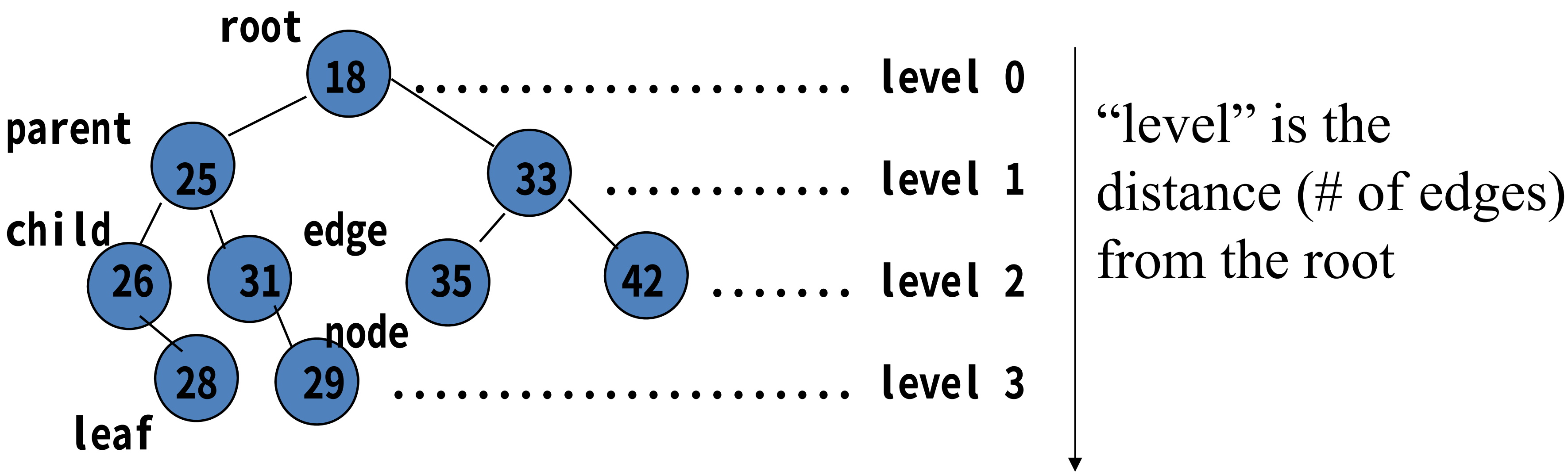
- Store: O(1)

```
heap[top++]=x
```

- Take: O(n)

```
m = 0;
for(i=1; i<top; i++)
  if(heap[i] < heap[m])
    m = i;
x = heap[m];
heap[m] = heap[top-1];
top = top - 1;
return x;
```

Heap by binary tree



root：node that has no parent

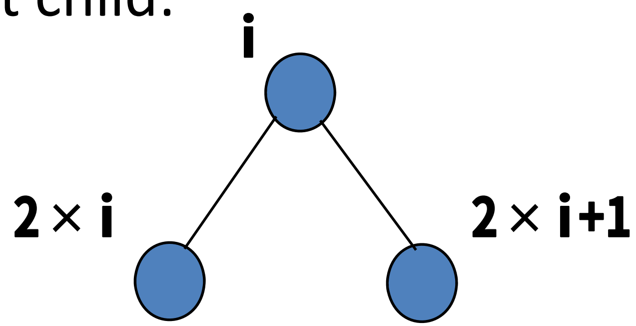
leaf：node that has no child

A tree is called a *binary tree*

if each node has at most 2 children

Property of binary tree for heap

1. Assign 1 to the root.
2. For a node of number i, assign 2 × i to the left child and assign 2 × i+1 to the right child:

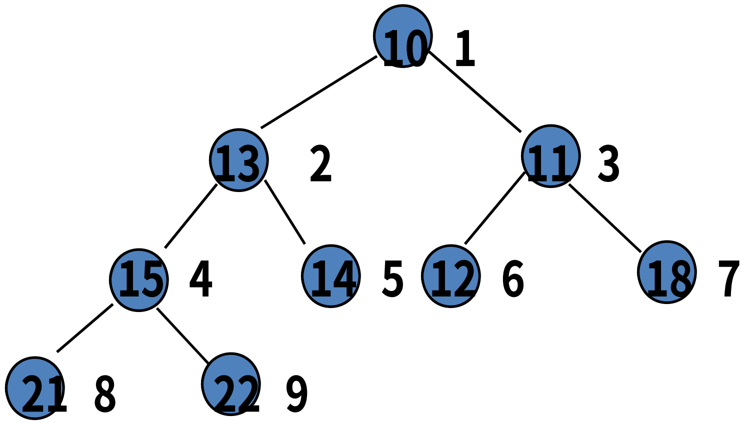


3. No nodes assigned by the number greater than n.
4. For each edge, parent stores data smaller than one in child.

The maximum level of heap: ceil($\log_2 (n+1) - 1$)

Each node has a unique path from the root, and its length is <u>O(log n).</u>

Example of a heap by binary tree



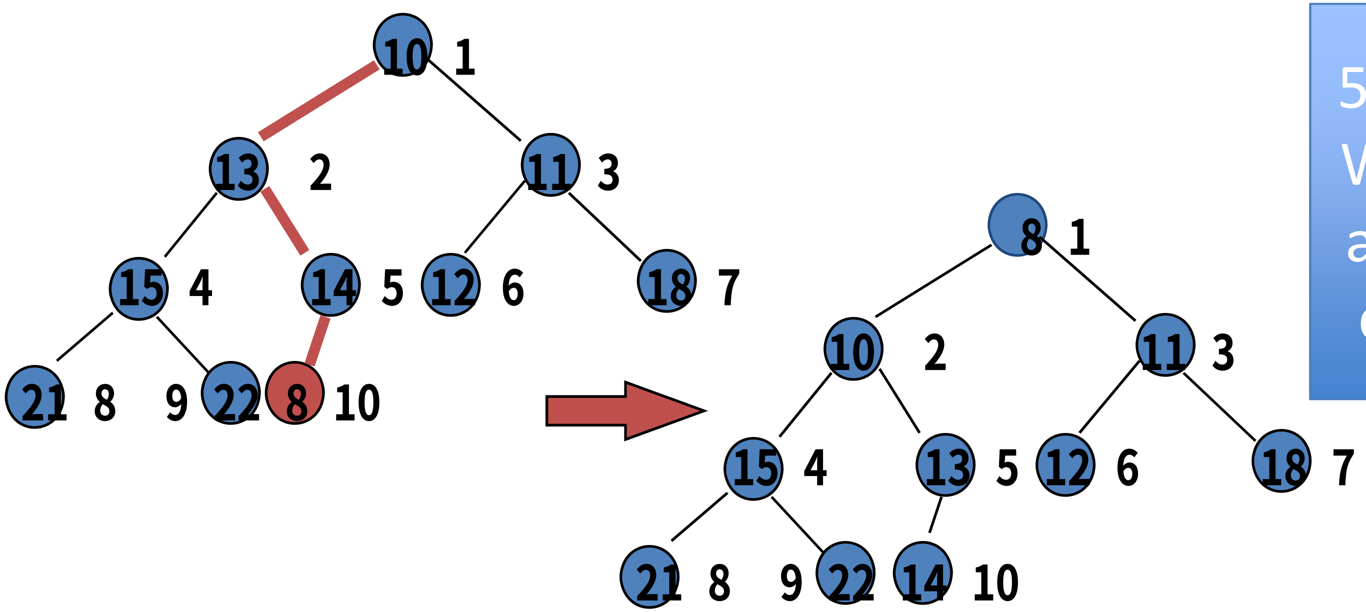
1. Assign 1 to the root.
2. For a node of number i, assign 2 × i to the left child and assign 2 × i+1 to the right child.
3. No nodes assigned by the number greater than n.
4. For each edge, parent stores data smaller than one in child.

We can use an array, instead of linked list!

1	2	3	4	5	6	7	8	9
10	13	11	15	14	12	18	21	22

Add a data to heap

(1) temporally, add data to node n+1 (n+1st element in array)
(2) traverse to the root step by step, and
if parent > child then swap parent and child

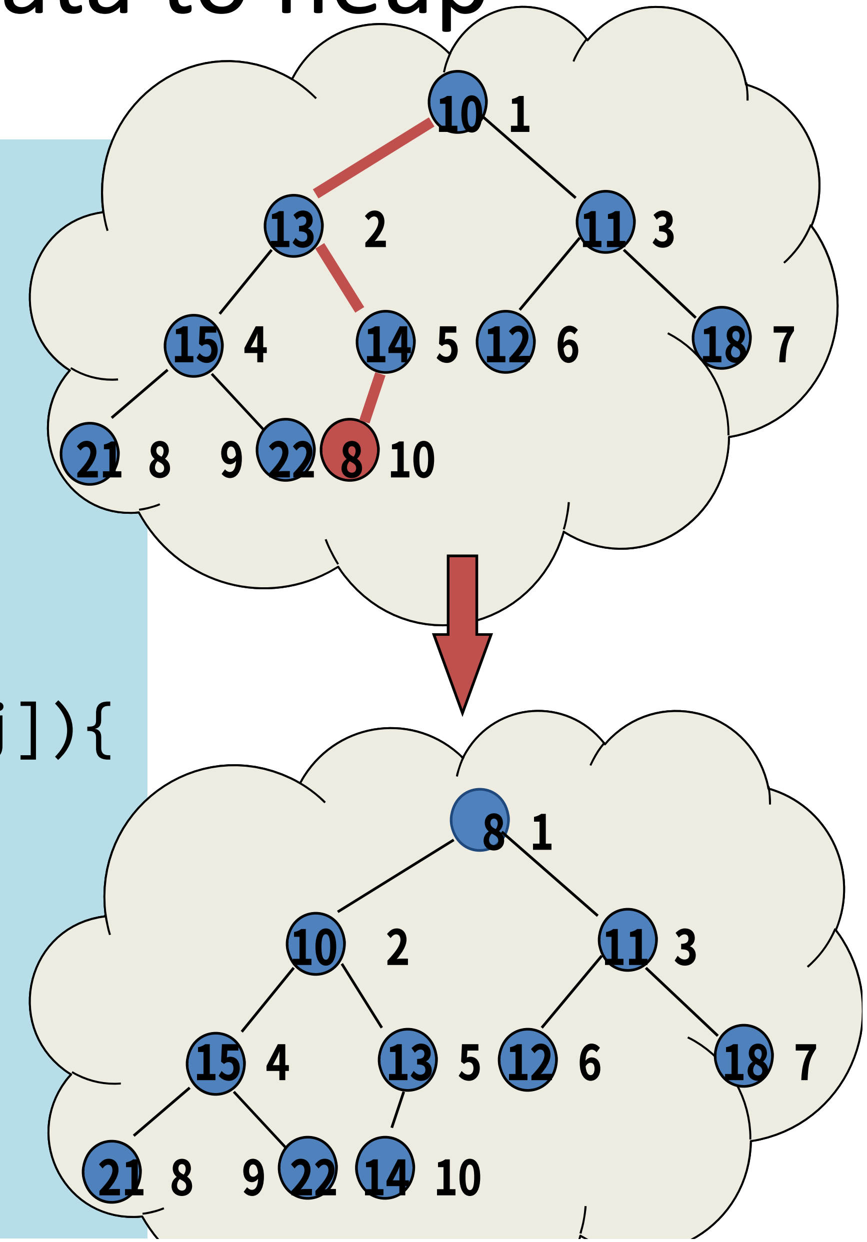


5 minute quiz: Why does this algorithm has consistency?

That is, from n+1st node to the root, the data are in order. This algorithm does not occur any problem with any other part of tree.

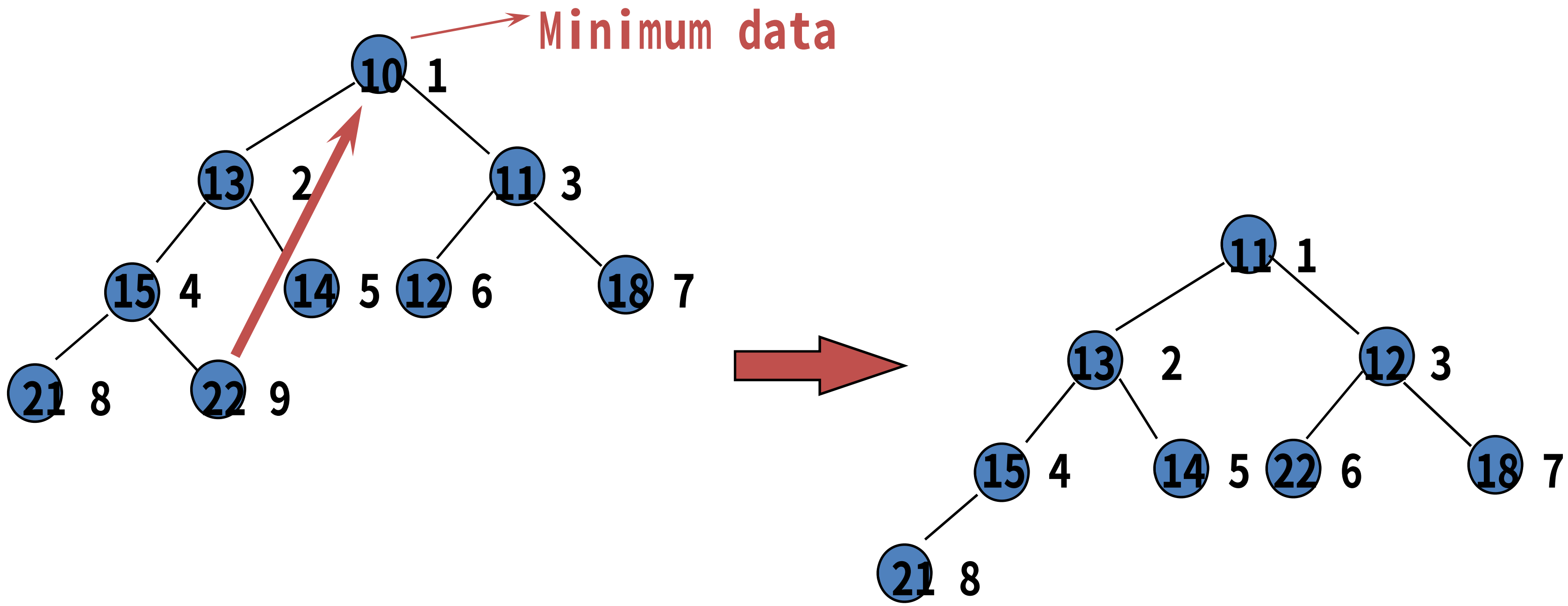
Program for adding a data to heap

```
void pushHeap(int x){
  int i, j;
  if(++n >= MAXSIZE)
    stop("Heap Overflow");
  else{
    heap[n] = x;
    i=n; j=i/2;
    while(j>0 && x < heap[j]){
      heap[i] = heap[j];
      i=j; j=i/2;
    }
    heap[i] = x;
  }
}
```



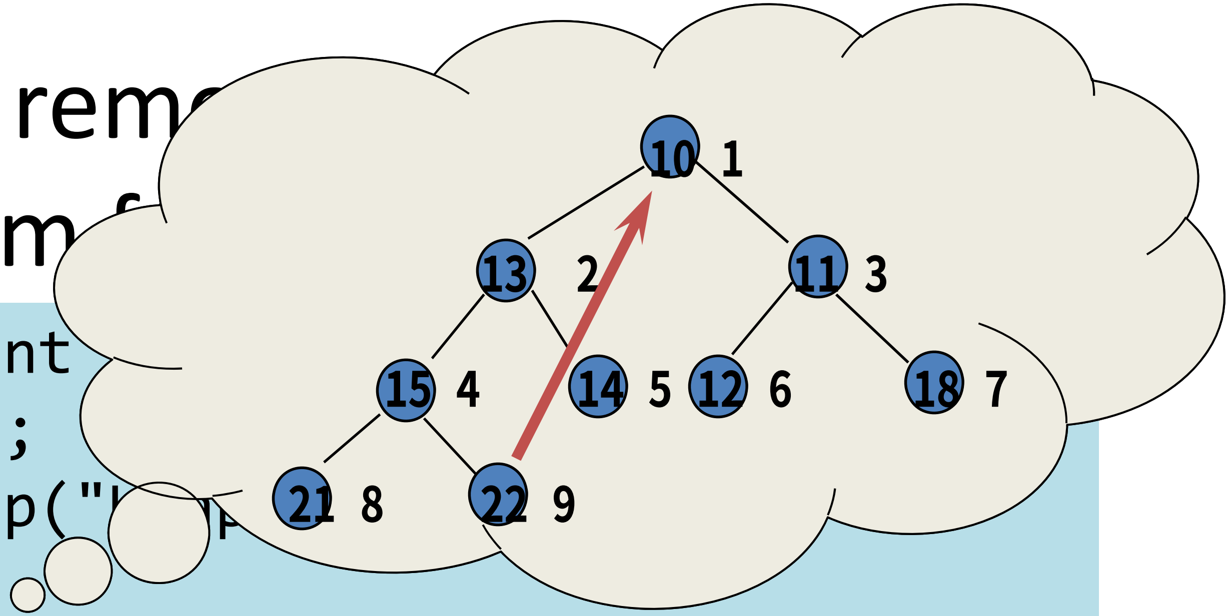
Heap: Take the minimum item

(1) Take the minimum data at the root
(2) Copy the last item (of number n) to the root
(3) Traverse from the root to a leaf as follows
For each pair of two children, choose the smaller one,
and exchange parent and child if child is smaller than parent.

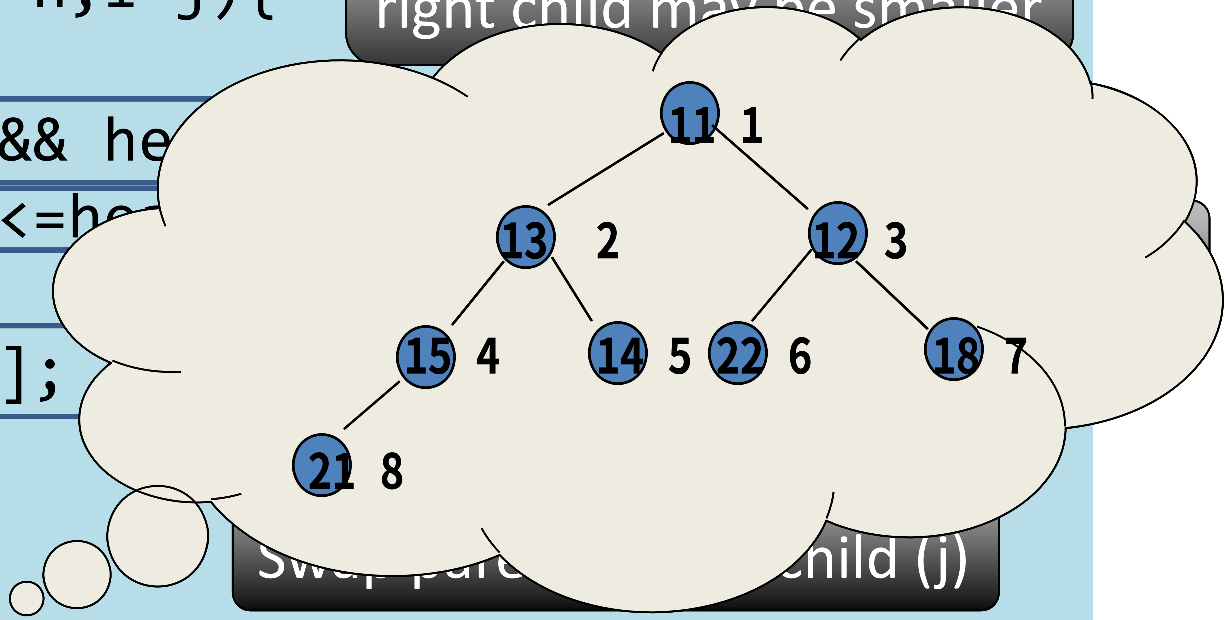


Program for rem
item f

```
int* deleteMin(int
  int x, i, j, t;
  if(n == 0) stop("
  else{
    heap[1]=heap[n--];
    for(i=1;i*2<=n;i=j){
      j=i*2;
      if(j+1<=n && he
      if(heap[i]<=he
      else {
        t=heap[i];
      }
  }}
  return heap;}
```



Node i has child &&
right child may be smaller



Swap pare … child (j)

Time complexity of binary heap

- Let n be the size of heap
  - Addition: O(log n)
  - Removal: O(log n)

- Each operation runs in time proportional to the depth of the heap
- The depth of heap is O(log n)