Introduction to Algorithms and Data Structures

Lesson 4: Searching (2) Block search

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Search Problem

- Problem: *S* is a given set of data. For any given data *x*, determine efficiently if *S* contains *x* or not.

- Efficiency: Estimate the time complexity by $n = |S|$, the size of the set *S*
  - In this problem, "checking every data in S" is enough, and this gives us an upper bound O(*n*) in the worst case.

Roughly, "the running time is proportional to *n*."

Data structure 2
Data in the array in increasing order

- s[]=

3	9	12	25	29	33	37	65	87

- This is something like dictionary and address book…

Q: Do you use sequential search algorithm when you check dictionary?

No!

Algorithm 2: m-block method

Idea of m-block method

(0) Divide the array into m blocks B_0, B_1, ... , B_{m-1}

(1) Check the biggest item in each block, and find the block B_j that can contain x

(2) Perform sequential search in B_j

Algorithm 2: m-block method

Idea of m-block method

(0) Divide the array into m blocks $B_0, B_1, \ldots, B_{m-1}$

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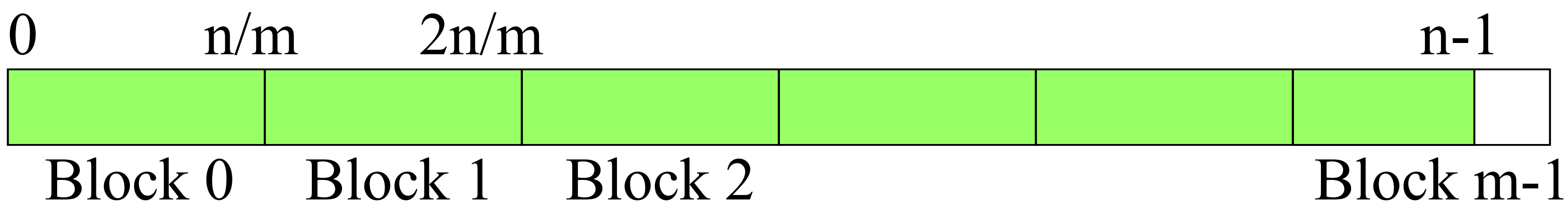
(2) Perform sequential search in B_j

Simple implementation:

divide into the blocks of same size except the last one.



- Each block has length k, where $k = \lceil n/m \rceil$
- Block B_j has items from s[jk] to s[(j+1)k-1]: $B_j = [jk, (j+1)k-1]$

Algorithm 2: m-block method

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(0) Divide the array into m blocks $B_0, B_1, \ldots, B_{m-1}$

(1) Check the biggest item in each block, and find the block B_j that can contain x

(2) Perform sequential search in B_j

```
j=0;
while(j<=m-2)
  if x<=s[(j+1)*k-1] then exit from loop
  else j=j+1;
```

If the program exits from the loop, the variable j indicates the index of the block, and j indicates the last one otherwise.

Algorithm 2: m-block method

<u>Idea of m-block method</u>

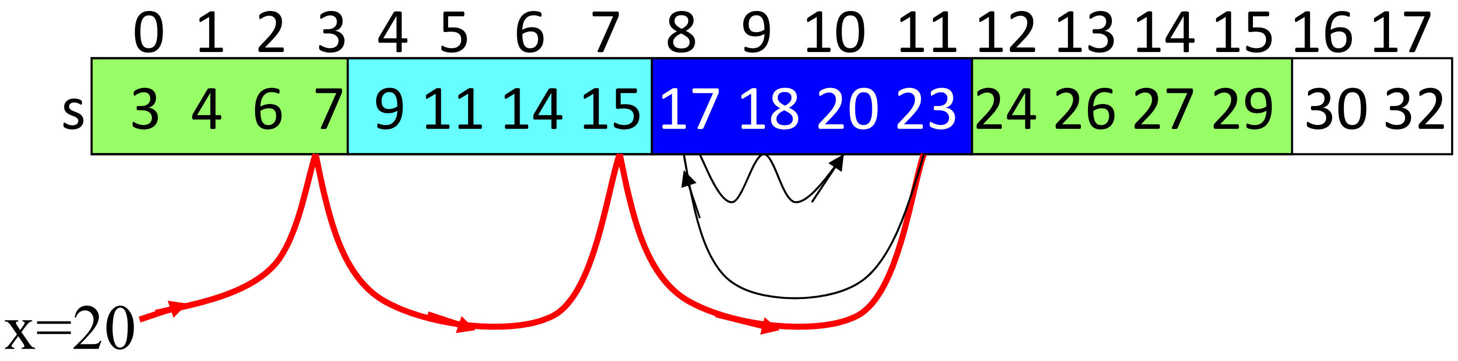
(0) Divide the array into m blocks $B_0, B_1, \ldots, B_{m-1}$

(1) Check the biggest item in each block,
and find the block B_j that can contain x

(2) Perform sequential search in B_j

```
i=j*k; t = min{ (j+1)*k-1, n-1 };
while( i < t )
  if x≧s[i] then exit from the loop;
  else i=i+1;  //next item in the block
if x == s[i] then return i and halt;
else  return -1 and halt.
```

Example and time complexity



- # of comparisons ≦ # of blocks ＋ length of block = m + n/m
- What the value of m that minimize m + n/m ?
  - Let f(m) = m + n/m, and take the differential for m
  - $f'(m) = 1 - n/m^2 = 0 \rightarrow m = \sqrt{n}$
  - When $m = \sqrt{n}$, # of comparisons ≦ $\sqrt{n} + n/\sqrt{n} = 2\sqrt{n}$
- Time complexity: $O(\sqrt{n})$

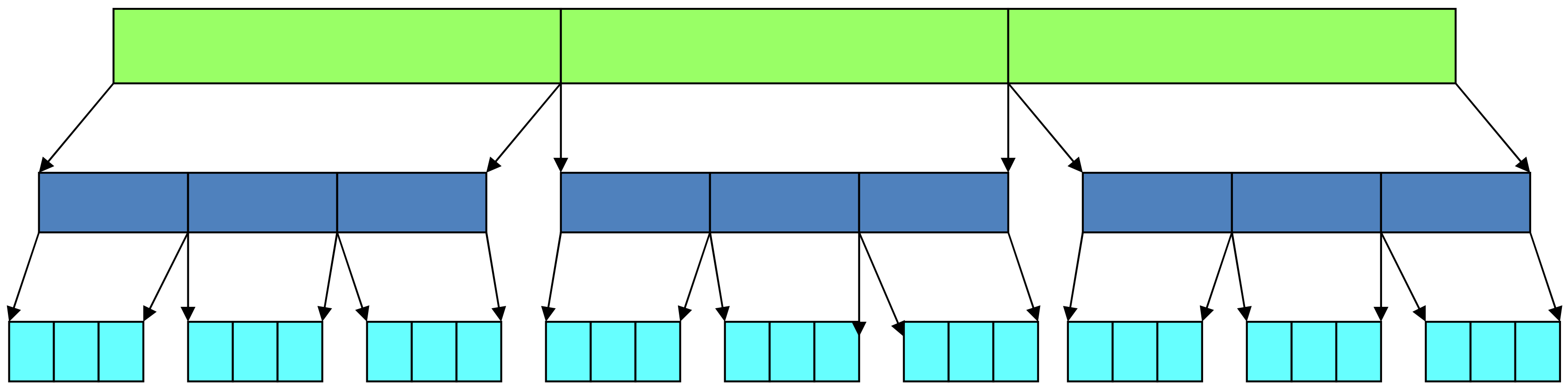
For example, when n=1000000,
Linear search takes n/2=500000 comparisons, but
Block search takes √1000000=1000 comparisons!!

Algorithm 3: Double m-block method

In the m-block method, we use sequential search in each block.

⇒ We can use m-block method again in the block!!

Recursive call: basic and **strong** idea



Idea of double m-block method

Divide search area into m blocks, and repeat the same process for the block that contains x, and repeat again and again up to the block has length at most some constant N

```
double-m-block-search(int left, int righ
    Length L = right − left + 1
    if  L > Lmin  then
        k = L/m;
        for  j = 0  to  m−2  do
            if  x ≦ s[left + (j+1)k − 1]  then  exit the loop;
        endfor
        f = left + jk;  t = min{left + (j+1)k − 1, n−1};
        double-m-block-search(f, t);
    else
        i = left;
        while (i < right)
            if  x ≦ s[i]  then  exit the loop  else i = i + 1;
        if  x  ==  s[i]  then  return i;
        else  return −1;
    endif
}
```

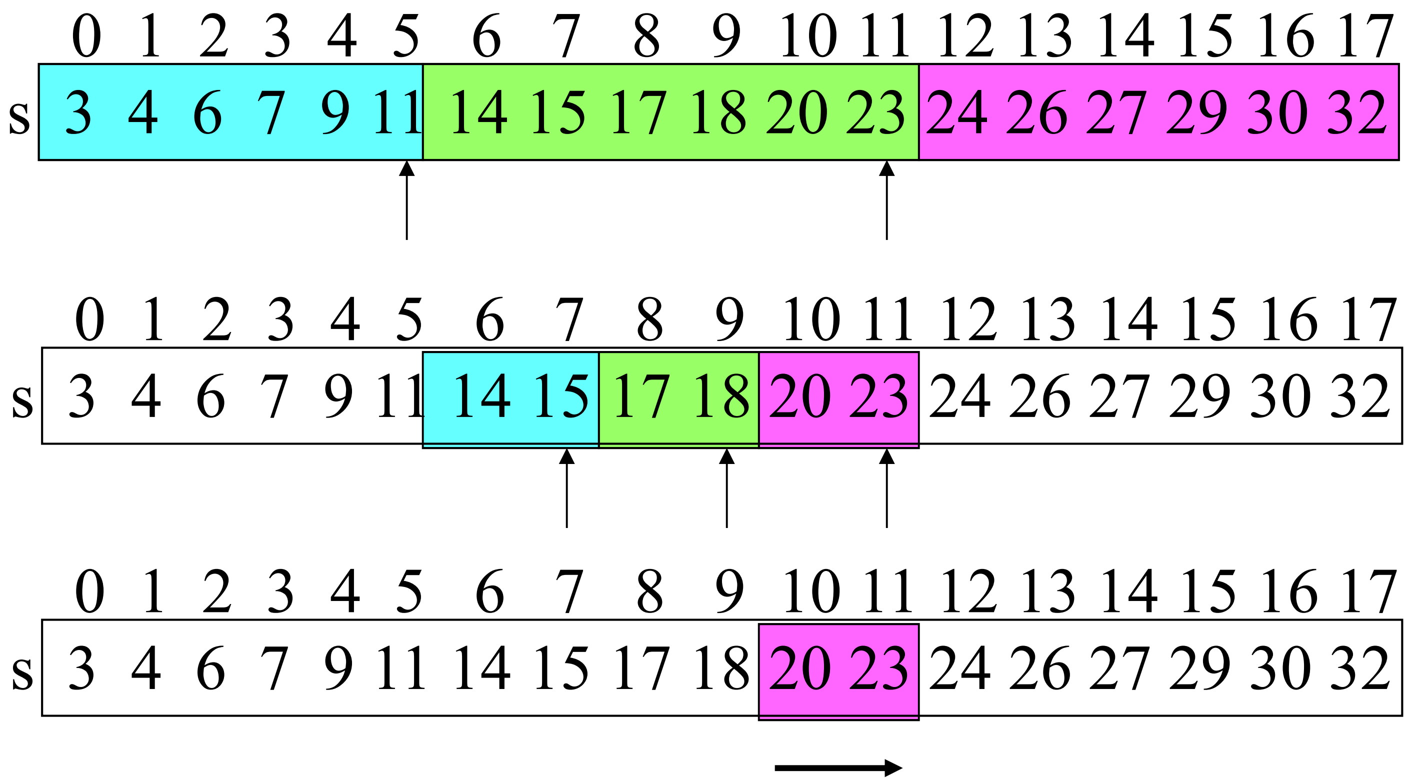
Some constant

Length of the block

Recursive call

sequential search if the interval is short enough

Example:
find 20 (x=20) for block size 3



Analysis of time complexity

- Length of search space

$$n \to \left\lceil \frac{n}{m} \right\rceil \to \left\lceil \frac{\left\lceil \frac{n}{m} \right\rceil}{m} \right\rceil \to \left\lceil \frac{\left\lceil \frac{\left\lceil \frac{n}{m} \right\rceil}{m} \right\rceil}{m} \right\rceil \to \cdots$$

- Let n_i be the length after the i-th call

$$n_1 = \left\lceil \frac{n}{m} \right\rceil \leq \frac{n}{m} + 1$$

$$n_2 = \left\lceil \frac{n_1}{m} \right\rceil \leq \frac{n}{m^2} + \frac{1}{m} + 1$$

$$\cdots$$

$$n_i \leq \frac{n}{m^i} + \sum_{j=0}^{i-1} \frac{1}{m^j} \leq \frac{n}{m^i} + 2$$

Analysis of time complexity

- The length n_i after the i-th recursive call:

  $n_i \leqq n/m^i + 2$

- How many recursive calls made?

$$n_i \leq \mathrm{Lmin} \Longleftarrow \mathrm{Lmin} \geq \frac{n}{m^i} + 2 \Longleftrightarrow i \geq \log_m \frac{n}{\mathrm{Lmin} - 2}$$

- Each recursive call make at most m-1 comparisons, so the total number of comparisons is $\leq (m-1)\log_m \frac{n}{\mathrm{Lmin}-2} + \mathrm{Lmin}$

- The time complexity is O(log *n*)

Analysis of time complexity: The best value of m

- $$T(n, m) = (m-1)\log_m \frac{n}{\mathrm{Lmin}-2} + \mathrm{Lmin}$$
$$= \frac{m-1}{\log_2 m}\log_2 \frac{n}{\mathrm{Lmin}-2} + \mathrm{Lmin}$$
- To make T(n,m) the minimum, smaller m is better because m-1 grows faster than $\log_2 m$ (which will be checked in the big-O notation).
- Therefore, m=2 is the optimal

We will have “binary search”