Reasoning in Nonclassical Logics

Hiroakira Ono

Japan Advanced Institute of Science and Technology

Formal reasoning

- Logics as formal sytems of reasoning.
- Logics as languages for describing structures and relations, and for modeling.

Thus, logic gives us a basic framework for formal description and verification.

First-order classical logic

- strong expressive power and generality, while undecidable.
- suitable as a language for describing mathematics, and rigorous arguments in general.

What are nonclassical logics, and what for?

Nonclassical reasoning

- constructive reasoning, relevant reasoning.
- resource-sensitive reasoning.
- reasoning with uncertain or incomplete information.
- reasoning about time and space.
- dynamic aspects, logic of action.
- epistemic aspects.
- reasoning about communication.

Nonclassical logics

- intuitionistic logic, relevant logics, linear logic.
- substructural logics.
- fuzzy logics, many-valued logics.
- temporal logics, logics of space.
- dynamic logics, berief revision.
- epistemic logics, common knowledge logics.

Why nonclassical logics?

- usually propositional logics.
- specific purpose vs all-purpose.
- trade-off between decidability (low complexity) and expressiveness.

Usually, these nonclassical logics are decidable. Somtimes, their expressive power is comparable with or even stronger than that of first-order classical logic.

We will focus mainly on:

1. computational aspects.
2. richness of expressive power.

- temporal logics.
- epistemic logics and common knowledge.

Temporal logics

Examples

- It will snow sometime in the future.
- He got a job but lost it later.
- Since I found a gap in my proof, I have always been nervous.

We introduce temporal operators, `F` at some future time, `P` at some past time, `S` since, `U` until and so on.

Temporal connectives and models

- `F` snow.
- `P` (he-loses-job `P` (he-gets-job)).
- `S`(I-find-gap, I-am-nervous).

Each temporal model is defined by using a flow of time `T,<` , where `<` is irreflexive, transitive and often linear.

Integers `Z,<` , Reals `R,<` , Rationals `Q,<` .

Decidability, Complexity

PTL: propositional temporal logic (with linear flows of time)

- PTL is finitely axiomatizable.
- Validity and satisfiability are decidable.

The complexity of satisfiability is:

- NP-complete if `U` , `S` omitted (Ono-Nakamura 1980 etc).
- PSPACE-complete if `U` , `S` included (Sistla-Clarke 1985 etc).

Expressive power

How expressive is temporal logic?

Each temporal formula can be translated into first-order logic. `S( , )` is translated as:

$$w\,(w < t_0 \quad (w)) \qquad u\,(w < u < t_0 \quad (u)))$$

How much of first-order logic can be covered by temporal formulas?

Expressive completeness

- Essentially, all of first-order formulas are expressed by temporal formulas with `U` and `S` in any linear flow of time without gap (Kamp 1968).

For example, `Z,<` , `R,<` , but not `Q,<` .

Second-order properties

Temporal formulas (and modal formulas in genral) can define some of second-order properties of flows of time.

For instance, it is known that well-orderedness (i.e. no infinite descending chains, or existence of the minimum element in each nonempty subset) is not a first-order property. Venema (1993) gave a finite axiom system for well-ordered flows of time, using the following formula:

$F \quad U(\ ,\neg\ )$.

Decidable fragments of predicate temporal logics.

Applications:

- specification and verification of programs, reactive systems.
- knowledge representation, etc.

Epistemic logics

An example (due to J. van Benthem)

Question. Is this the road to Kanazawa? (I)
Answer. Yes. (You)

- What information has passed?

I know it, but I didn't know it before.

Epistemic statements

Moreover,

You know that I know it.
I know that you know I know it.

Various epistemic states can be described.

- I know your password.
- I know that you don't know I know your password.

K_I vs $K_I \neg K_U K_I$

Common knowledge

Now, both this is the road to Kanazawa and I know this is the road to Kanazawa are common knowledge between you and I.

Common knowledge is more than what all of us know.

- $K_I \quad K_U$.
- $K_U K_I\ , K_I K_U K_I\ , K_U K_I K_U K_I\ , \ldots$

Common knowledge logics are decidable.

Logic and communication

- From logic of single agent to logic of multi-agents

  Many agents, and various kinds of information flow and action in computational process.

- computer science.
- game theory, economics.

Research ongoing

- modal, temporal and epistemic logics — with King's College, London etc.
- theorem provers for basic modal logics — with "Logic and Computaion" of NICTA.
- substructural logics, including many-valued logics, fuzzy logics and relevant logics — Project: $\mathtt{A}_{\mathtt{L}}$ (Algebra & Substructural Logics).
- algebraic methods in nonclassical logics — Barcelona Meeting 2005.