The Focus Problem: A Fundamental Issue in Automatic Verification

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Outline

Background

- automatic proof search
- the focus problem
- an example

The Soft-SCOTT algorithm

- the SCOTT project
- using soft constraints

Experimental results

- some algebraic problems
- the set theory example

Conclusion and future work

Automated deduction

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   Search in space of sequents for provable subgoals.

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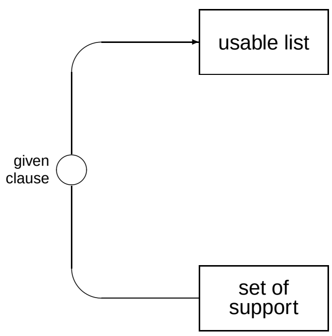

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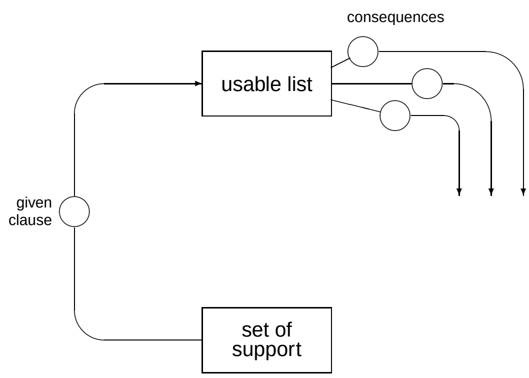

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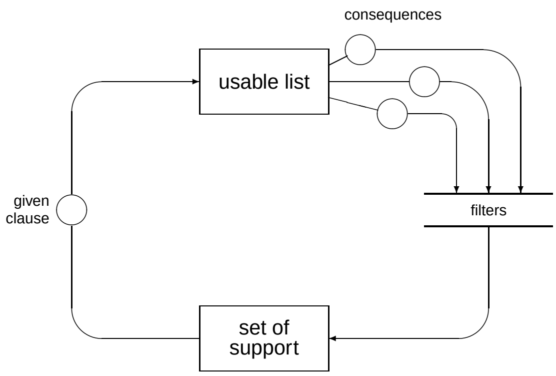

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Software certification

First order provers now powerful enough to be used for software certification in industry

- SafeLogic (Sweden)
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General technique

- Use e.g. Hoare to reduce to small proof obligations
- Prove these without human intervention
- Require extensions e.g. for numbers
- Most are easy, a few are hard

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Sources:

- John Harrison (INTEL)
- David Crocker (Escher)
- Bernd Fischer (NASA)

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  - etc.

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Exhibits focus problem

Simple examples e.g. $x \cap y = y \cap x$ too hard for OTTER

Results

plain OTTER without any guidance

topic focus OTTER with term weighting to make it prefer clauses about $\cap$ to clauses about $\cup$ or $\emptyset$ etc

formula focus OTTER with topic focus plus a weighting scheme to make it prefer clauses containing actual subterms of the goal

	$x \cap y = y \cap x$			$x \cap y \subseteq y \cap x$		
	plain	**topic**	**formula**	**plain**	**topic**	**formula**
iterations	—	—	128	766	350	66
clauses generated	—	—	1729	12742	6593	1018
time (seconds)	—	—	0.2	4.4	1.3	0.1

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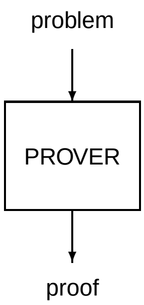
4. So if M makes most of the usable list true, and c is in the set of support, it is good to take c as the next given clause.

In fact we don't know whether c is inconsistent with S, but if we choose a clause that is false in M we have a better chance than if we choose arbitrarily.

SCOTT Architecture



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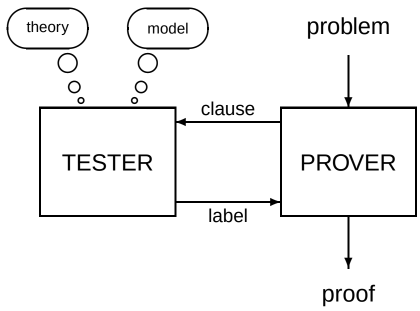

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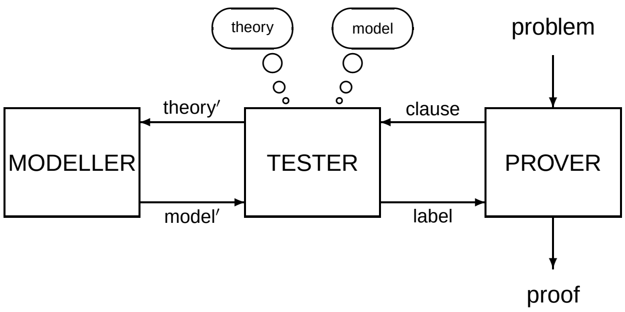

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Third phase 2003–2004

Single approximate model instead of many exact ones.

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- For SCOTT, treat any initially usable clauses as hard and all later activated clauses as soft
- Gives an *approximate* model of *all* of the usable list rather than an *exact* model of just *part* of it
- Gains speed because only one model, and robustness because all usable clauses modelled together regardless of activation order

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- Time in model generator versus quality of guidance

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In a loop, $\forall xyz[(x(yz))x = (xy)(zx)]$ implies $((ab)c)b = a(b(cb))$

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Statistics

	with models	without
Input clauses	20	20
Clauses generated	3149	397803
Clauses kept	1649	30179
Clauses given	57	587
Clauses in proof	36	—
Models generated	13	0
Time	4.28 sec	600.65 sec

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Statistics

	with models	without models
Input clauses	38 (61)	38 (61)
Clauses generated	56831	129125
Clauses kept	27071	21709
Clauses given	184	249
Clauses in proof	25	25
Models generated	142	0
Time	417.44 sec	3.01 sec

Results on set theory problem

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without guidance	**plain**	**topic**	**formula**	**plain**	**topic**	**formula**
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with guidance	**plain**	**topic**	**formula**	**plain**	**topic**	**formula**
iterations	—	3009	169	496	241	85
clauses generated	—	80239	2430	9576	3520	1426
time (seconds)	—	90.0	3.2	6.7	2.4	0.6

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- Applications (software certification?)